Homework #11, EECS 451, W04. Due **Fri. Apr. 16**, in class

- This is the last graded homework.
- This may not be graded and returned before the third exam, so you may wish to photocopy it.
- Final exam information will be posted on the web site.

Skill Problems

1. [B 10] Text 6.11. Concept(s): ***DFT via FFT.***
Do decimation in time only. You may use MATLAB's FFT command to check your results.

2. [B 10] Text 6.17. Concept(s): ***Samples of z-transform around circle of radius r***

3. [B 10] Text 6.19. Concept(s): ***DFT of DFT.***
Hint: use `fft(fft( ))` in MATLAB to check your formula.

4. [B 20] Concept(s): ***Inverse FFT from FFT.***
Use the results of text problem 6.19 to write a MATLAB m-file called `my_ifft.m` that computes the N-point inverse FFT of any given input vector. Your m-file should use MATLAB's `fft` routine and some simple operations, but not `ifft` of course. This should be about a 3 line `m`-file.
This problem illustrates the fact that if you have a working FFT routine available, you can easily create an inverse FFT routine from it.
Hint: check your function vs MATLAB's `ifft` to make sure it is correct.
Hint: `a = [100:109]; b = a([10 9:-2:1 1])` will return (try it): `b = [109 108 106 104 102 100 100]`

5. [B 30] Concept(s): ***FFT of real sequences.***
Use the technique in Section 6.2.1. to write a MATLAB m-file `fftconvreal.m` that (linearly) convolves two real sequences of arbitrary length using as few FFT calls as possible. This can be done in about a half dozen lines of code. Hint: check the operation of your function by comparing with MATLAB's `conv` for some simple sequences.

Mastery Problems

6. [B 10] Text 8.6. Concept(s): ***FIR design by frequency sampling.***
Plot both $h[n]$ and $|\mathcal{H}(\omega)|$.

7. [B 10] Concept(s): ***sampling rate conversion***
A signal $x_{\mathrm{a}}(t)$ was anti-alias filtered and sampled at rate $F_1 = 10$kHz, and the samples $x[n]$ stored. Sometime later, it is desired to playback this signal using an D/A converter that only runs at the sampling rate $F_2 = 15$kHz. If you put $x[n]$ into that D/A converter, the output signal will playback too fast. We need to upsample $x[n]$ by a factor of $15/10$, but that ratio is not an integer!
One way to solve this problem is to use a combination of upsampling and downsampling as follows:

$$x[n] \to \boxed{\uparrow 3} \to \boxed{\mathcal{H}_1(\omega)} \to \boxed{\downarrow 2} \to \boxed{\mathcal{H}_2(\omega)} \xrightarrow{y[n]} \boxed{\text{D/A, } F_2 = 15\text{kHz}} \to \hat{x}_{\mathrm{a}}(t)\,.$$

Specify the magnitude responses of the filters $\mathcal{H}_1(\omega)$ and $\mathcal{H}_2(\omega)$ so that the final output signal $\hat{x}_{\mathrm{a}}(t)$ will be as close to the original signal $x_{\mathrm{a}}(t)$ as possible.