Homework #8, EECS 451, W04. Due **Fri. Mar. 19**, in class

Notes

- Exam 2 is Fri. Mar. 26 in class (50 minutes). Go to the *opposite* room as last time.
- It will cover Chapters 2-4, 8.1, 8.2, and HW 1-8. (The material is inherently cumulative.)
- This HW will not be returned before the exam. Photocopy your answers for comparison with the posted solutions.
- Guaranteed exam topics include: sampling, up/down sampling, pole-zero plots, frequency response.
- You may bring a calculator to the exam, and two 8.5x11" pieces of paper with your own handwriting on both sides. Tables of common transforms will not be provided on the exam.

You will use MATLAB on almost all of these HW problems. This is how real signal processing work is done!

Skill Problems

1. [B 20] Concept(s): ***Filtering "hum" from an audio recording.***
A rare Jimi Hendrix recording was recently re-discovered at a recording studio. However, the (former) sound-engineer did not protect against EM interference. Load `jimi.mat` from the usual place and listen to this short excerpt from the recording[a]. If your headphones have adequate low-frequency response, you should hear an annoying 120 Hz hum. (The sampling rate is 8000Hz.)
(a) [20] Design a filter that will remove the 120 Hz contamination without degrading the remaining audio signal excessively. Show the pole-zero plot and the magnitude reponse of the filter you design.
(b) [0] Use MATLAB's `filter` command to filter the signal, and listen to the result. Repeat (a) if needed.

[a] Your parents may enjoy learning that you are listening to Hendrix for a homework assignment...

2. [B 40] Text 8.1a-c. Concept(s): ***FIR design using windows.***
Also show the pole-zero plot for each design. Hint: `tf2zp` may be useful.
Just turn in 8 plots: impulse response, magnitude response (in dB), phase response, and pole-zero plot for each of the two windows. Use `subplot(421)` etc. to save paper. As always, label all axes.

3. [B 10] Text 8.9. Concept(s): ***FIR design of digital "differentiator."***

4. [B 5] Concept(s): ***Filter design tradeoffs.***
Here are 3 choices one must make in filter design.
   - 1. FIR vs IIR
   - 2. large M vs small M
   - 3. rectangular window vs non-rectangular window

   Here are possible tradeoffs associated with those choices.
   - A. width of transition band vs sidelobe amplitude
   - B. width of transition band vs time lag and computation
   - C. linear phase vs nonlinear phase

   Match each choice with the corresponding tradeoff. Just turn in the number,letter pairs.

Mastery Problems

5. [B 20] Text 8.25. Concept(s): ***bandlimited signal processing.***
For part (b), use MATLAB to plot the impulse response and magnitude response of your filter.

6. [B 15] Concept(s): ***sampling, down-sampling, up-sampling.***
An audio signal with spectrum $X_a(F) = 1 - |F|/15\text{kHz}$ for $|F| \le 15\text{kHz}$ is sampled at 40kHz after passing through an ideal anti-alias filter. To reduce storage, you digitally down-sample the DT signal by a factor of two. Later, to play the signal back, you upsample it again by a factor of two (inserting zeros). Then you pass the upsampled signal through an ideal D/A converter.
   (a) [10] Carefully sketch the magnitude spectrum of the signal that comes out of the sound card. How does it compare to the original signal spectrum?
   Hint: consider spectrum at each step, graphically when easier.
   (b) [5] Explain how to modify the downsampling and upsampling procedures so that the final output signal better matches the input signal.

7. [B 30] Concept(s): ***2D DTFT and image filtering.***
The 2D DTFT of a 2D signal (*i.e.*, image) $x[n,m]$ is defined as follows:

$$X(\omega_1, \omega_2) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} x[n,m]\, \mathrm{e}^{-\jmath(\omega_1 n + \omega_2 m)},$$

and the inverse 2D DTFT is:

$$x[n,m] = \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} X(\omega_1, \omega_2)\, \mathrm{e}^{\jmath(\omega_1 n + \omega_2 m)}\, \mathrm{d}\omega_1\, \mathrm{d}\omega_2.$$

   (a) [0] Prove the following shift property of the 2D DTFT:

$$x[n - n_0, m - m_0] \overset{\text{2D DTFT}}{\leftrightarrow} \mathrm{e}^{-\jmath(\omega_1 n_0 + \omega_2 m_0)}\, X(\omega_1, \omega_2)$$

   (b) [0] 2D convolution is defined by

$$h[n,m] ** x[n,m] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} h[n-k, m-l]\, x[k,l].$$

   Prove the following convolution property of the 2D DTFT:

$$h[n,m] ** x[n,m] \overset{\text{2D DTFT}}{\leftrightarrow} H(\omega_1, \omega_2) X(\omega_1, \omega_2).$$

   (c) [10] For a 2D filter with impulse response $h[n,m]$, its 2D DTFT $H(\omega_1, \omega_2)$ is called the frequency response. Find the frequency response of the 2D filter having impulse response

$$h[n,m] = \frac{1}{5} \left(\delta_{2D}[n,m] + \delta_{2D}[n-1,m] + \delta_{2D}[n+1,m] + \delta_{2D}[n,m-1] + \delta_{2D}[n,m+1]\right),$$

   where the 2D Kronecker impulse is $\delta_{2D}[n,m] = \delta[n]\, \delta[m]$.
   Simplify $H(\omega_1, \omega_2)$ as much as you can.
   (d) [0] Does this filter look like a lowpass or highpass filter?
   (e) [20] Apply this filter to a real image using the MATLAB mfile `dtft2_intro1.m` on the web site.
   (f) [0] From looking at the images, do you think this filter is lowpass or highpass? Why?