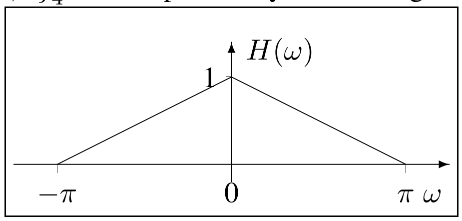
Homework #6, EECS 451, W04. Due **Fri. Mar. 5**, in class

Review Problems

R1. [B 0] Concept(s): ***DTFS and filters***

The periodic signal $x[n] = \{\underline{1}, 0, 2, 0\}_4$ is the input to a system having the following frequency response.



(a) [0] Sketch the spectrum of the input signal $x[n]$.

(b) [0] Sketch the spectrum of the output signal $y[n]$.

(c) [0] Express the output signal $y[n]$ as a sum of sinusoids.

R2. [B 0] Concept(s): ***Orthogonality of complex exponentials.***

(a) [0] Prove the equality

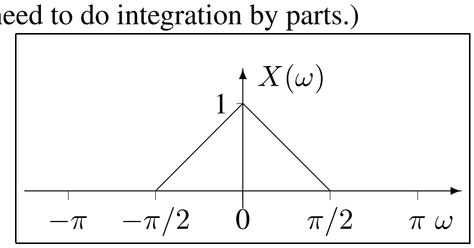
$$\frac{1}{N} \sum_{n=0}^{N-1} e^{\jmath \frac{2\pi}{N} kn} = \begin{cases} 1, & k = 0, \pm N, \pm 2N, \ldots \\ 0, & \text{otherwise.} \end{cases}$$

(b) [0] Show that the harmonically related complex exponential signals $s_k[n] = e^{\jmath \frac{2\pi}{N} kn}$ are orthogonal over any interval of length N, *i.e.*, $\sum_{n=n_0}^{n_0+N-1} s_k[n]\, s_l^*[n] = 0$ if $k \neq l$.
(Thus the DFT is an orthogonal transformation.)

Skill Problems

1. [B 20] Concept(s): ***Inverse DTFT.***

(a) [10] Find the signal $x[n]$ that has the following spectrum. (You may use symbolic integration software if you want; otherwise you may need to do integration by parts.)



Hint: you can (and should) simplify your final answer so that it involves sinc^2.

(b) [0] Observe that the signal $h[n] = \delta[n-1] + \delta[n+1]$ has spectrum $H(\omega) = 1\, e^{-\jmath\omega} + 1\, e^{\jmath\omega} = 2\cos(\omega)$.

(c) [10] Find the signal $y[n]$ that has the spectrum $Y(\omega) = \cos^2(\omega)$. Do *not* perform integration; instead use "coefficient matching" by considering the previous problem.

2. [B 10] Concept(s): ***DTFT "differencing" and "integration" properties.***

Find the relationship between $Y(\omega)$ and $X(\omega)$ for the following time-domain relationships.

(a) [5] $y[n] = x[n] - x[n-1]$. (Differencing, analogous to differentiation in continuous time.)

(b) [5] $y[n] = \sum_{k=-\infty}^{n} x[k]$ (Accumulation, analogous to integration in continuous time.)

3. [B 0] Concept(s): ***DTFT properties.***

A signal $x[n]$ has the DTFT $X(\omega) = \frac{1}{1 - a e^{-\jmath\omega}}$. Find the DTFT of the following signals.

(a) [0] $e^{\jmath \frac{\pi}{2} n}\, x[n]$

(b) [0] $\cos(0.3\pi n)\, x[n]$

(c) [0] $x[n] * x[n-1]$

(d) [0] $x[n] * x[-n]$

4. [B 20] Concept(s): ***Inverse systems.*** $H(\omega)$ ***to pole-zero.***
A system $\mathcal{T}$ has frequency response $H(\omega) = 1 - e^{-j\omega} + \frac{2}{9}\,e^{-j2\omega}$.
(a) [10] Find the pole-zero plot for $\mathcal{T}$.
(b) [0] Using that plot, make a rough sketch by hand of the magnitude response.
(c) [10] Determine the impulse response of the corresponding *inverse system* $\mathcal{T}^{-1}$.
Do not use the DTFT synthesis integral to solve this!

5. [B 20] Concept(s): ***sampling and spectra.***
Consider the analog signal $x(t)$ with the following spectrum:

$$X(F) = \begin{cases} 2, & |F| \leq 0.5\text{kHz} \\ 1, & 0.5\text{kHz} < |F| < 1.25\text{kHz} \\ 0, & \text{otherwise.} \end{cases}$$

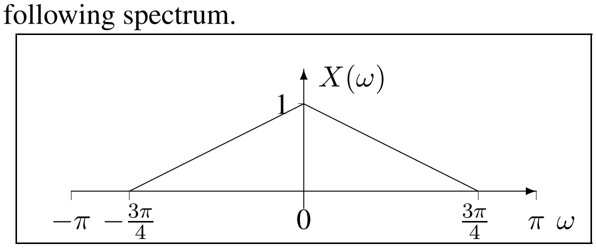
This signal is sampled at $F_s = 2000$Hz to form a DT signal $x[n]$. Find $X(\omega)$ and $x[n]$.

6. [B 15] Concept(s): ***sampling and signal recovery.***
(a) [10] Find the continuous-time signal $x(t)$ that when sampled (without aliasing) will produce a signal $x[n]$ having spectrum $X(\omega) = 7\pi\,\delta(\omega - \pi/4) + 7\pi\,\delta(\omega + \pi/4)$. Assume the sampling rate F_s is 10kHz.
(b) [5] Find a *different* continuous-time signal $x_2(t)$ that, when sampled at the same rate, yields the same samples $x[n]$ due to aliasing.

Mastery Problems

7. [B 40] Concept(s): ***up sampling and down sampling.***
Suppose $x[n] \overset{\text{DTFT}}{\leftrightarrow} X(\omega)$. Express the DTFT of the following signals in terms of $X(\omega)$.
(a) [10] Upsampling by zero insertion: $y_1[n] = \begin{cases} x[n/2]\,, & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$
(b) [10] Downsampling (by discarding): $y_2[n] = x[2n]$. Hint. Think about $\frac{1+(-1)^k}{2}$.
(c) [10] Upsampling by repeating: $y_3[n] = \begin{cases} x[n/2]\,, & n \text{ even} \\ x[(n-1)/2]\,, & n \text{ odd} \end{cases}$
(d) [10] Odd zeroing: $y_4[n] = \begin{cases} x[n]\,, & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

8. [B 40] Concept(s): ***Up sampling and down sampling and DTFT.***
The signal $x[n]$ that has the following spectrum.



Sketch the spectra of the signals $y_1[n]$, $y_2[n]$ and $y_4[n]$ defined in the previous problem over the interval $[-\pi, \pi]$.