Homework #3, EECS 451, W04. Due **Fri. Jan. 30**, in class

Notes

- Exams 1 and 2 *will* be held during class time, using rooms 1001 and 1005 EECS, on the dates in the syllabus.

Review Problems

R1. [B 0] Find the z-transforms of these signals.

(a) $x_1[n] = \{3, 0, 0, 0, 0, \underline{6}, 1, -4\}$

(b) $x_2[n] = \begin{cases} (1/2)^n, & n \geq 5 \\ 0, & \text{otherwise} \end{cases}$

(c) Sketch the pole-zero plot for $x_2[n]$.

R2. [B 0] Text 3.6. Concept(s): ***time summation property.***
State the analogous property for continuous-time signals using Laplace transforms.

R3. [B 0] Concept(s): ***convolution and time shifts.***
One of following two statements is correct, and the other is incorrect. The symbol $\boxed{*}$ denotes *convolution*.

- If $y[n] = h[n] * x[n]$ then $y[n-3] \stackrel{?}{=} h[n-3] * x[n-3]$
- If $y[n] = h[n] * x[n]$ then $y[n-3] \stackrel{?}{=} h[n] * x[n-3]$

(a) [0] Give a simple proof of the correct statement.

(b) [0] Give a simple counter-example for the incorrect statement.

(c) [0] Repeat (a) and (b) for the following two statements. The symbol $\boxed{\cdot}$ denotes *multiplication*.

- If $y[n] = h[n] \cdot x[n]$ then $y[n-3] \stackrel{?}{=} h[n-3] \cdot x[n-3]$
- If $y[n] = h[n] \cdot x[n]$ then $y[n-3] \stackrel{?}{=} h[n] \cdot x[n-3]$

(d) [0] Think about the difference between the two operations. (Not graded.)

Be careful with the notation "$h[n] * x[n]$." More precise notation is "$(h * x)[n]$," which makes it clear that convolution is an operation on two *signals*, whereas the notation "$h[n]*x[n]$" can be confused as an operation on just two numbers.

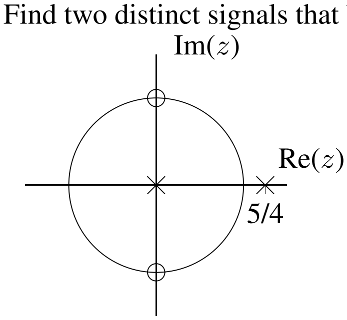
R4. [B 0] Concept(s): ***Plotting*** $|X(z)|$
Use MATLAB to plot $|X(z)|$ for the signal $x[n] = (1/2)^n \, u[n]$.

Skill Problems

1. [B 30] Concept(s): ***practical experience with the cross correlation.***
Certain new Mercedes Benz models have cruise control systems that sense the distance to the next car in front and slow down if necessary. Naturally, BMW wants to emulate this in their next models, so they have hired you as a consultant. BMW has decided to use forward looking Radar to detect the distance to the car in front of you. From the Web site you can download the MATLAB file `bmw.mat`, containing two signals $x[n]$ and $y[n]$. The signal $x[n]$ is (samples of) the radar signal that is transmitted from your car. The signal $y[n]$ is the radar signal that is reflected off the car in front of you. The signal $y[n]$ is a delayed version of $x[n]$, and there is noise added to $y[n]$, *i.e.*, $y[n] = x[n - n_0] + \text{noise}[n]$ for some integer delay n_0. Your job is to determine how far away is the car in front of you. Samples of the two signals are provided from $n = 0$ to $n = 50$. Assume each sample corresponds to a distance of 0.2 meters.
   (a) [10] Stem plot the two signals $x[n]$ and $y[n]$ using `subplot` to arrange them vertically.
   (b) [0] Try to determine the delay n_0 by eye.
   (c) [20] Use the cross correlation of the two signals to estimate the delay n_0.
   The MATLAB command `xcorr` makes this very easy. See `help xcorr`.
   Hint: if the input signals to `xcorr` are sampled from $n = 0$ to $n = 50$, then the output of `xcorr` is the cross correlation $r_{xy}[l]$ for $l = -50$ to $l = 50$. Plot the output of `xcorr` vs the appropriate indices, then find the maximum using the `max` command. (Note: the `max` command has two output arguments.)

2. [B 40] Concept(s): ***finding z-transforms and pole-zero plots.***
Find the z-transforms of the following discrete-time signals. As usual, include the ROC.
   (a) [10] $x_1[n] = (1 + n)\, u[n]$
   (b) [10] $x_2[n] = \cos\left(\frac{\pi}{2}n - \frac{\pi}{3}\right) u[n-3] + \delta[n-2]\, u[n]$. (Use no decimals in your answer.)
   (c) [10] $x_3[n] = \sum_{k=-\infty}^{n} 3^k\, u[n]$
   (d) [10] Sketch the pole-zero plot for $X_2(z)$.

3. [B 10] Concept(s): ***region of convergence***
Find two distinct signals that both have the following pole-zero plot. Hint: one is not causal.



4. [B 15] Concept(s): ***computing a z-transform.***
Consider $x[n] = \{\underline{1}, 0, -1, 0, 1, 0, -1/5, 0, 1, 0, -1/25, \ldots\}$ (the pattern repeats indefinitely).
   (a) [5] Determine $X(z)$. Hint: express $x[n]$ as the sum of two signals.
   (b) [10] Use MATLAB to make a plot of $|X(z)|$. Make sure your ranges cover the interesting aspects.
   Hint: see answer to a review problem.

5. [B 15] Concept(s): ***system function from block diagram***
Find the system function $H(z)$ for the system shown in Fig. P8.24 on p. 734.

Mastery Problems

6. [B 40] Concept(s): ***up sampling and down sampling.*** These are very important signal processing operations. Suppose $x[n] \overset{Z}{\leftrightarrow} X(z)$. Express the z-transform of the following signals in terms of $X(z)$.

   (a) [10] Upsampling by zero insertion: $y_1[n] = \begin{cases} x[n/2]\,, & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

   (b) [10] Downsampling: $y_2[n] = x[2n]$. Hint. Think about $\frac{1+(-1)^k}{2}$.

   (c) [10] Upsampling by repeating: $y_3[n] = \begin{cases} x[n/2]\,, & n \text{ even} \\ x[(n-1)/2]\,, & n \text{ odd} \end{cases}$

   (d) [10] Odd zeroing: $y_4[n] = \begin{cases} x[n]\,, & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

7. [G 0] Concept(s): ***LTI systems and invertibility.***
   Prove that if an LTI system is invertible, then the inverse system is LTI.