EECS 401, Midterm 2, 6/12/97, 1-2:30 PM

1. A random variable X has the following probability density function:

$$f_X(x) = \begin{cases} c/x^2, & 1 \le x \le 2 \\ 0, & \text{otherwise.} \end{cases}$$

- Find c.
- Find the cdf of X.
- Find the standard deviation of X.
- Find $P[|X - 1| < 1/2]$.
- Let $Y = \ln X$. Find the pdf of Y.

2. In a binary communication system a 0 bit is transmitted by applying a 0 volt input to a noisy communication channel, whereas a 1 bit transmitted by applying a 1 volt input to the channel. The output voltage Y is the sum of the input voltage plus a noise voltage N, i.e., $Y = X + N$ where X is either 0 or 1 volts. The noise voltage N is a random variable with the following triangular pdf:

$$f_N(z) = \begin{cases} 1 - |z|, & |z| \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

Assume 0 and 1 bits are equally likely. Hint: sketch the pdfs.

- Find $P[Y > 1]$.
- Find $E[Y/2]$
- If we observe that the output voltage Y is between 1/2 and 1 volts, then what is the probability that a 0 bit was transmitted?

3. X and Y are random variables with the following joint probability density function:

$$f_{X,Y}(x, y) = \begin{cases} 1/x^2, & 1 \le x \le 2 \text{ and } 1 \le y \le 3 \\ 0, & \text{otherwise.} \end{cases}$$

- Find the marginal pdf of Y. What is the name of the distribution?
- Are X and Y independent? Explain why or why not. (Be complete.)
- Find $P[X + Y < 3]$.

end