Solutions to EECS 401, Spring 1997, Midterm 1, May 23, 1997

1.

- $P(\bar{B}) = P(\{4,5,6\}) = 0.48$
- $P(A - B) = P(\{4,6\}) = 0.39$
- $P(\bar{B}|A) = P(\bar{B} \cap A)/P(A) = P(A - B)/P(A) = 0.39/0.6 = 0.65$
  Several students wrote $P(\bar{B}|A) = P(\bar{B} \cap A)/P(\bar{B})$. Not true!
- example: C = {4,5} and D = {6}.
- No. $P(A \cap B) = P(\{2\}) = 0.21 \neq P(A)P(B) = 0.6 \cdot 0.52 = 0.312$

2.

- 6 heads in 15 tosses: $\boxed{\binom{15}{6}/2^{15}} = 5005/2^{15} \approx 0.1527$
- 8 heads in 8 tosses: $\boxed{1/2^8}$
- A = {6 H in 10 tosses}, B = {1 H in 5 tosses}, $P(A \cap B) = P(A)P(B) = \boxed{\binom{10}{6}\binom{5}{1}/2^{15}}$
- one sequence out of 2^15: $\boxed{1/2^{15}}$
- A_1 = {first 10 tosses show heads (rest are anything)}, $P(A_1) = 1/2^{10}$.
  B = {10 heads in 15 tosses}. Note $A_1 \cap B$ = {first 10 tosses heads, last 5 tosses tails}.
  $P(A_1|B) = P(A_1 \cap B)/P(B) = (1/2^{15})/\binom{15}{10}/2^{15} = \boxed{1/\binom{15}{10}}$
  Second approach is to use same B as above, but let A_2 = {first 10 tosses show heads, last 5 tosses are tails}, so $P(A_2) = 1/2^{15}$. But $A_2 \cap B$ = {first 10 tosses heads, last 5 tosses tails}, as before. So $P(A_2|B) = P(A_2 \cap B)/P(B) = (1/2^{15})/\binom{15}{10}/2^{15} = 1/\binom{15}{10}$.
  Common error was to confuse A_1 and A_2 when using Bayes rule for this problem, because $P(B|A_2) = 1$, but $P(B|A_1) = 1/2^5$.

3.

- By total probability: $P(B) = P(B|W)P(W) + P(B|D)P(D) = 0.01 \cdot 0.98 + 0.99 \cdot 0.02 = \boxed{0.0296}$
- By Bayes rule: $P(D|B) = P(B|D)P(D)/P(B) = 0.99 \cdot 0.02/P(B) = 0.99 \cdot 0.02/0.0296 = \boxed{0.6689}$
- $P[X < 5]$ = P(0 to 4 bad out of 20) = $\boxed{\sum_{k=0}^{4} \binom{20}{k} (P(B))^k (1 - P(B))^{20-k}}$
  where $P(B) = 0.0296$ from first part.
  Many students worked out the value even though the instructions said that was unnecessary.

The abbreviation "SMW" on your exam means you must Show More Work to get full credit (or more partial credit). If you would like more elaboration on these solutions, please come to office hours.

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Percentile      25%     50%     75%
Combined        77      89.5    97
Scores:
A 100 100 100 100 100 98 98 98 98 98 98 97 97 97 95 94 93 92 91 91 91
B 88 87 85 84 83 80
C 79 78 77 76 69 69 64 61
D 57 54 52 35 34
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