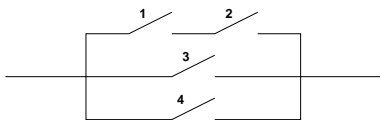


1.

- If 2% of the cars that BMW makes are lemons and your dealership buys 100 cars, find the probability that 2 or more of the cars you buy will be lemons.
- The switches in the following network are independent. Each switch is closed with probability  $p$ . Find the probability that there is a closed path between the input and output.



2. A *skilled* dart thrower tosses darts at a square dartboard. The coordinates  $X$  and  $Y$  of where the dart lands are random variables with the following joint probability density function:

$$f_{X,Y}(x,y) = \begin{cases} \frac{9}{16}(1-x^2)(1-y^2), & |x| \leq 1, |y| \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

- Find the marginal pdf of  $X$ .
- One random variable of interest here is the distance  $Z$  of the dart from the center of the dartboard, i.e.  $Z = \sqrt{X^2 + Y^2}$ . Find  $E[Z^2]$ .
- Find the pdf of  $W = 2 \cdot \max\{X, Y\}$ .

3. Ursula rolls a fair *4-sided* die (a tetrahedron) 600 times. Virgil tosses a fair coin 600 times. Assume all tosses and rolls are independent. Wilma counts up the total number of dots and heads. Call the sum  $W$ .

- Find an exact expression for the probability that  $W$  takes the value 601.  
Hint: consider  $S_W$ .
- Find an approximation for the probability that  $W$  is less than 1845.  
Hint: form the sum by adding up the dots and heads in pairs:  $W = \sum_i (U_i + V_i)$ .

4. A RAID (redundant array of disks) system consists of  $n$  disk drives connected in parallel to provide maximum redundancy, where  $n > 1$ . The lifetime of each disk drive is an exponential random variable with mean failure time of 1 year. The RAID system fails when *all* of the  $n$  disk drives fail. Let  $T$  denote the failure time of the RAID system.

- A customer asks you to configure a RAID system for which the probability of system failure within the first year is less than  $\alpha = 0.05$ . Determine how many disk drives are required.
- You also sell this customer a monitor. The lifetime of your monitor is an exponential random variable  $X$  with mean 2 years. The customer also buys a monitor from a competitor of yours; the lifetime of that monitor is an exponential random variable  $Y$  with mean 3 years. Find the probability that your monitor will fail before the competitor's monitor. Assume the two monitor's failure times are independent.

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