EECS 401, Final, 6/23/97, 8-10:00 AM

1.

- Let X be the number of lemons $P[X \geq 2] = 1 - P[X = 0] - P[X = 1] = 1 - (0.98)^{100} - 100(0.98)^{99}(0.02) = 1 - (0.98)^{99}(2.98) \approx 0.597$

- $P[C_4 \cup C_3 \cup (C_1 \cap C_2)] = 1 - P[C_4^c \cap C_3^c \cap (C_1 \cap C_2)^c] = 1 - (1-p)^2 P[(C_1 \cap C_2)^c] = 1 - (1-p)^2(1 - P(C_1 \cap C_2)) = \boxed{1 - (1-p)^2(1-p^2) = 2p - 2p^3 + p^4} = 1 - q^3(2-q)$
  Many students incorrectly used additivity and got $p^2 + 2p$ even though the C_i's are not *disjoint (4 points).*

2.

- For $|x| \leq 1$: $f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy = \int_{-1}^{1} \frac{9}{16}(1-x^2)(1-y^2)\,dy = \boxed{\begin{cases} \frac{3}{4}(1-x^2), & |x| \leq 1 \\ 0, & \text{otherwise.} \end{cases}}$

- $E[Z^2] = E[X^2 + Y^2] = E[X^2] + E[Y^2]$. $E[X^2] = \int_{-1}^{1} \frac{3}{4} x^2(1-x^2)\,dx = (3/4)(2/3 - 2/5) = 1/5$.
  Thus $\boxed{E[Z^2] = 2/5.}$

- Note $S_W = [-2, 2]$. Note $F_X(x) = \int_{-1}^{x} \frac{3}{4}(1-t^2)\,dt = \frac{3}{4}(t - \frac{1}{3}t^3)\big|_{-1}^{x} = \frac{3}{4}x - \frac{1}{4}x^3 + \frac{1}{2}$.
  $F_W(w) = P[W \leq w] = P[2\max\{X,Y\} \leq w] = P[X \leq w/2, Y \leq w/2] = F_X(w/2)F_Y(w/2)$. Thus $f_W(w) = \frac{1}{2}f_X(w/2)F_Y(w/2) + F_X(w/2)\frac{1}{2}f_Y(w/2) = f_Y(w/2)F_X(w/2) = \boxed{\begin{cases} \frac{3}{4}(1-(w/2)^2)(\frac{3}{4}(w/2) - \frac{1}{4}(w/2)^3 + \frac{1}{2}), & |w| \leq 2 \\ 0 & \text{otherwise.} \end{cases}}$

3.

- Note: $S_W = \{600, 601, \ldots, 3000\}$, but outcomes are *not* equally likely.
  P[W=601] = P[{roll 600 ones} $\cap$ {toss 1 head and 99 tails}] + P[{roll 599 ones and 1 two} $\cap$ {toss 100 tails}] $= \frac{1}{4^{600}} \cdot 600\frac{1}{2^{600}} + 600\frac{1}{4^{600}} \cdot \frac{1}{2^{600}} = \boxed{1200/8^{600} \approx 1.68 \cdot 10^{-539}.}$

- Let U_i be the number of dots on i roll, let V_i be one of the ith toss is heads, and 0 otherwise. Then $W = \sum_{i=1}^{600} X_i$ where $X_i = U_i + V_i$.
  $E[X] = E[U] + E[V] = 2.5 + 0.5 = 3$. So $E[W] = 600 \cdot 3 = 1800$.
  $E[U^2] = (1^2 + 2^2 + 3^2 + 4^2)/4 = 15/2$, so $\text{Var}\{U\} = 15/2 - (2.5)^2 = 30/4 - 25/4 = 5/4$. *Many errors here.*
  $\text{Var}\{X\} = \text{Var}\{U\} + \text{Var}\{V\} = 5/4 + 1/4 = 3/2$. So $\sigma_W = \sqrt{n}\sigma_X = \sqrt{600 \cdot 3/2} = 30$.
  $P[W < 1845] = P[(W - 1800)/30 < (1845 - 1800)/30] \approx \boxed{1 - Q(1.5) \approx 0.9332}$.

4.

- We need $P[T \leq 1] = P[T_1 \leq 1, \ldots, T_n \leq 1] = (1 - e^{-1})^n \leq \alpha$. So $n \log(1 - e^{-1}) \leq \log\alpha$ or $n \geq \log\alpha / \log(1 - e^{-1}) \approx 6.53$.
  Thus $\boxed{n = 7}$ disk drives are required.

- By independence, $f_{X,Y}(x,y) = f_X(x)f_Y(y) = (e^{-x/2}/2)(e^{-y/3}/3)$ for $x, y > 0$.
  We want $P[X < Y] = \int_{-\infty}^{\infty}\int_{-\infty}^{y} f_{X,Y}(x,y)\,dx\,dy = \int_0^{\infty}\int_0^{y}(e^{-x/2}/2)(e^{-y/3}/3)\,dx\,dy = \int_0^{\infty}(e^{-y/3}/3)(1 - e^{-y/2})\,dy = 1 - \frac{1}{3}\int_0^{\infty} e^{-y5/6}\,dy = 1 - \frac{2}{5} = \boxed{3/5.}$

If you would like more elaboration on these solutions, please come to office hours.

Percentile	25%	50%	75%
Combined	35	52.5	72

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(38 students)  Scores:
100 98 93 92 90 87 83 76
73 71 70 66 65 64 62 60 59
57 53 52 52 50 49 48 46 45 42 42
35 35 35 34 31 28 26 19 19 8
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end