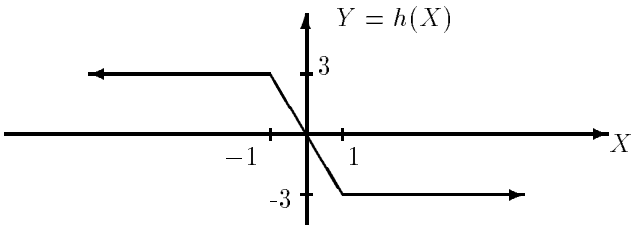
EECS 401, Winter 1996, Midterm 2, Mar 20, 1996

1. Assume X is a random variable having the shifted-Laplacian pdf

$$f_X(x) = \frac{1}{2} e^{-|x-5|}.$$

Define $Y = h(X)$, where $h(x)$ is the clipped scaling inverter:

$$h(x) = \begin{cases} 3, & x < -1 \\ -3x, & |x| \le 1 \\ -3, & x > 1 \end{cases}$$



- Find $P[|X-5| \le 3]$
- Are the events $[|X-5| \le 3]$ and $[X > 5]$ independent? Explain why or why not.
- Find and sketch the probability density function for Y.

2. A bin contains 25% resistors supplied by company A, and 75% supplied by company B. The resistances of those supplied by company A are random variables uniformly distributed between 1 and 4 Ohms. Those supplied by company B are uniformly distributed between 1 and 2 Ohms.
A resistor is selected at random from the bin, and exactly 15 volts is applied to it.
(Let R denote the resistance.)

- Find the probability that the current exceeds 5 amps.
  Note: Ohm's law is $V = IR$ (voltage equals current (in amps) times resistance (in Ohms)).
- If the power (W) exceeds 75 watts (power in watts is V^2/R), find the probability that the chosen resistor was supplied by company A.
  (To avoid confusion, please use the letter W to denote power rather than the letter P.)
- Find the variance of R.

3. A 2-meter glass rod drops and breaks into two pieces. The break point X has the triangular pdf

$$f_X(x) = \begin{cases} 1 - |x-1|, & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}.$$

- Find the probability that the ratio of the length of the shorter piece over the length of longer piece is less than 1/2.
- Find the expected value of the product of the lengths of the two pieces.

end