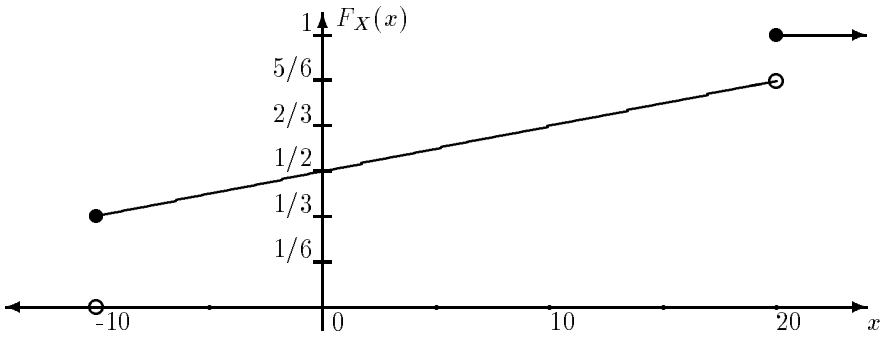
EECS 401 Midterm 1, Winter 1996

1. Assume X is a random variable with the cumulative distribution graphed below.

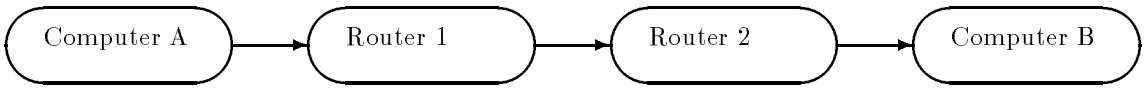


Define the following events:

$$A = [X \leq 0], \quad B = [|X| < 10], \quad C = [-5 \leq X \leq 10], \quad D = [X \text{ is a multiple of } 5]$$

- Find $P(\bar{B})$
- Find $P(A \cup C)$
- Find $P(C - A)$
- Find $P(D)$
- Find $P(A|C)$

2. Computer A is connected to Computer B through two routing devices, as pictured below. When Computer A sends a message to Computer B, the message is separated into packets. The first packet is a header that includes information on how many total packets are in the message. This first packet always reaches Computer B. However, due to network traffic, either of the two routers may fail to pass along some of the subsequent data packets, i.e., a router may "drop" packets. When Computer B receives too few packets, it repeatedly requests retransmission (of the entire message) until the complete message is received.
Assume the probability that Router 1 drops one or more data packets is q_1.
Assume the probability that Router 2 drops one or more data packets is q_2.
Assume the routers drop packets independently.



- What is the probability the message will reach Computer B on the first attempt?
- What is the probability that fewer than 13 attempts are required for successful transmission of the message?

3. A glass rod drops and breaks into 2 pieces.
Assume the break point is equally likely to occur anywhere along the rod.

- Find the probability that the ratio of the length of the shorter piece over the length of longer piece is less than 1/4.
- Now suppose the glass rod breaks into three pieces.
  Write down a computer program (in any programming language) that uses the Monte Carlo method to compute an approximation to the probability that the longest piece is at least 3 times longer than the shortest piece.
  (The concept is more important than precise syntax or elegance.)