Name: _____
ID Number: _____

EECS 401 Final
Apr. 25, 1996

!! KEEP THIS PAGE FACE-UP UNTIL YOU ARE TOLD TO BEGIN !!

- This is an closed book exam. **No books or calculators are permitted**. You are permitted to use **one page of notes** (8-1/2 by 11 inch, both sides) all of which must be in your own handwriting.
- There are 3 problems, worth a total of 100 points. The questions may not be in order of increasing difficulty; read all before beginning.
- Box your final answer. You will be graded on both the final answer and the steps leading to it. Correct intermediate steps will help earn partial credit. For full credit, ~~cross out~~ any incorrect intermediate steps.
- The following integrals may be useful:
  $\int_0^\infty \frac{1}{a} e^{-x/a}\, dx = 1$
  $\int x^m \log x\, dx = x^{m+1}\left[\frac{\log x}{m+1} - \frac{1}{(m+1)^2}\right]$ for $m \neq -1$
- Simplify your result when possible, but you need not compute factorials.
- Specify all ranges when giving density or distribution functions.
- This exam has 2 pages. Make sure your copy is complete.
- Write the engineering honor pledge on your exam below and sign.
  (I have neither given nor received aid on this exam.)

1. (30 points)
A large bin contains an equal number of red (2 Ohm) and blue (6 Ohm) resistors. You select 100 resistors at random from the bin and form a chain by linking them in series (so the resistances add).

- [10 points] Find the expected value of the total resistance of the chain.
  Hint: let X_i denote the resistance of the ith resistor selected.
- [10 points] Find an *exact* expression for the probability that the total resistance of the chain takes a value exceeding 430 Ohms.
  Hint: if one selects N blue resistors, then the remaining $100 - N$ are red resistors; express the total resistance in terms of N.
- [10 points] Find a numerical value that closely approximates the probability that the total resistance of the chain exceeds 430 Ohms.

2. (20 points)
The voltage Y applied across a resistor is a random variable with pdf $f_Y(y) = e^{-y}u(y)$. The resistance X is a random variable with pdf $f_X(x) = 2e^{-2x}u(x)$. Assume that X and Y are independent random variables. Define Z to be the current across the resistor: $Z = Y/X$.

- [10 points] Find $E[2(X-1)^7(Y-1)+3]$
- [10 points] Find $f_Z(z)$.

3. (50 points)
Let X_i, $i = 1, 2, \ldots$ be a sequence of i.i.d. Gaussian random variables with mean 2 and variance 9. Define the sum process $S_n = \sum_{i=1}^{n} X_i$ for $n = 1, 2, \ldots$.

- [10 points] Find $C_S(50, 60)$.
- [10 points] Define another random process Y_i by $Y_i = X_i^2$, $i = 1, 2, \ldots$.
  Is Y_i strict-sense stationary, wide-sense stationary, neither, or both? Explain.
- [10 points] Find $E[S_{52}|S_{50} = 6]$.
- [10 points] Find the probability that S_{100} takes a value less than 260.
- [10 points] Find the correlation coefficient of S_{64} and X_7.

end