1. Let X and Y be random variables with $\bar{X} = 2$, $\bar{Y} = 3$, $\sigma_X^2 = 5$, $\sigma_Y^2 = 20$.
The correlation coefficient between X and Y is $-1/2$.

- [2 points] Determine $E[2 - 3X + 4Y]$.
- [2 points] Determine $\mathrm{Var}[3X + 1]$.
- [2 points] Determine $E[2Y^2]$.
- [3 points] Determine the covariance between X and Y.
- [3 points] Determine the correlation between X and Y.
- [2 points] Are X and Y statistically independent? Explain why or why not.
- [3 points] Could there be real constants a and b such that $Y = aX + b$? Explain why or why not.
- [3 points] Determine $\mathrm{Var}[X - Y]$.

2. Let X and Y be independent random variables. Assume Y is uniformly distributed on the interval [1,5]. Also assume

$$f_X(x) = \begin{cases} x/16, & 1 \le x \le 3 \\ 0, & \text{otherwise} \end{cases} .$$

- [3 points] Determine the probability density function for Y.
- [7 points] Determine the joint probability density function for X and Y.
- [8 points] Determine $P(XY \le 4)$.
- [6 points] Determine $E[X^2 Y]$.

3. Let X be a continuous random variable with cumulative probability distribution function $F_X(x)$. Define a new random variable Y in terms of X by $Y = |X| + 2$.

- [7 points] Express the cumulative probability distribution function for Y in terms of the cumulative probability distribution function for X.
- [7 points] Assume X has a uniform distribution on the interval $[-2, 1]$. Determine the density function $f_Y(y)$ and sketch it.

4. For some detection systems, the effect of noise is multiplicative, and there is also a random DC shift (such as dark current in a photo-transistor). Let $X(t)$ be the input signal, $N(t)$ be the noise process, and A be the random DC shift.
For your purposes, simply assume the output signal is expressed in terms of the inputs by:

$$Y(t) = A + X(t)N(t).$$

Also assume the following:

- $X(t)$ is a wide-sense stationary random process with mean $\bar{X} = 2$ and with known autocorrelation function $R_{XX}(\tau)$.
- $N(t)$ is a wide-sense stationary random process with mean $\bar{N} = 5$, and with known autocorrelation function $R_{NN}(\tau)$.
- A is a random variable with a uniform distribution on [0,6].
- The random processes $X(t)$ and $N(t)$ and the random variable A are all statistically independent.

- [3 points] Determine $E[A]$ and $E[A^2]$.
- [4 points] Determine $E[Y(t)]$.
- [8 points] Is $Y(t)$ wide-sense stationary? If not, explain why not; if it is, explain why.
- [7 points] Now assume $R_{XX}(\tau) = 4 + 2e^{-|\tau|}$ and $R_{NN}(\tau) = 25 + 3e^{-|\tau|}$. Determine σ_Y^2.

5. Your company stocks diodes in two boxes. Diodes in box B_1 have a random lifetime with an exponential distribution with mean $m_1 = 5$ years. Diodes in box B_2 have a random lifetime with an exponential distribution with mean $m_2 = 10$ years. Note that the density and distribution functions for a random variable Y with an exponential distribution having mean m are:

$$\begin{aligned} f_Y(y) &= \frac{1}{m} e^{-y/m}\ u(y) \\ F_Y(y) &= (1 - e^{-y/m})\ u(y). \end{aligned}$$

You select one of the two boxes at random, and pick a diode from the selected box. Let X denote the (random) lifetime of the selected diode.

- [4 points] Determine the distribution function for X.
- [4 points] Determine the expected lifetime of the selected diode.
- [4 points] Determine the probability that the lifetime of the selected diode exceeds 20 years.
- [8 points] Pose your own probability question related to this diode problem. Your question must require Bayes' Rule to solve. Write down and circle your question, then solve for the probability using Bayes' Rule. Hint: your question will probably be of the form: "What is the probability that ________________ given that ________________ ?"