Image Compression

or
How to squeeze lots of pictures into an iPod

Jeffrey A. Fessler

EECS Department
The University of Michigan

Engin 110

2008-10-09

Alternative Energy - Follow Up

Winter 2009 Course: EECS 498 (Special Topics)
Grid Integration of Alternative Energy Sources
Prof. Ian Hiskens

- Photovoltaics (solar cells)
- Power electronic interfaces
- Fuel cells
- Power systems
- Wind generation, including basic electrical machines
- Plug hybrid electric vehicles

Prerequisite: electrical circuits (*e.g.*, EECS 215 or EECS 314)

Electrical engineering will be the heart and brain of all alternative energy solutions

- heart: circulatory system (energy distribution)
- brain: central nervous system (information / control)

- energy / power
- information

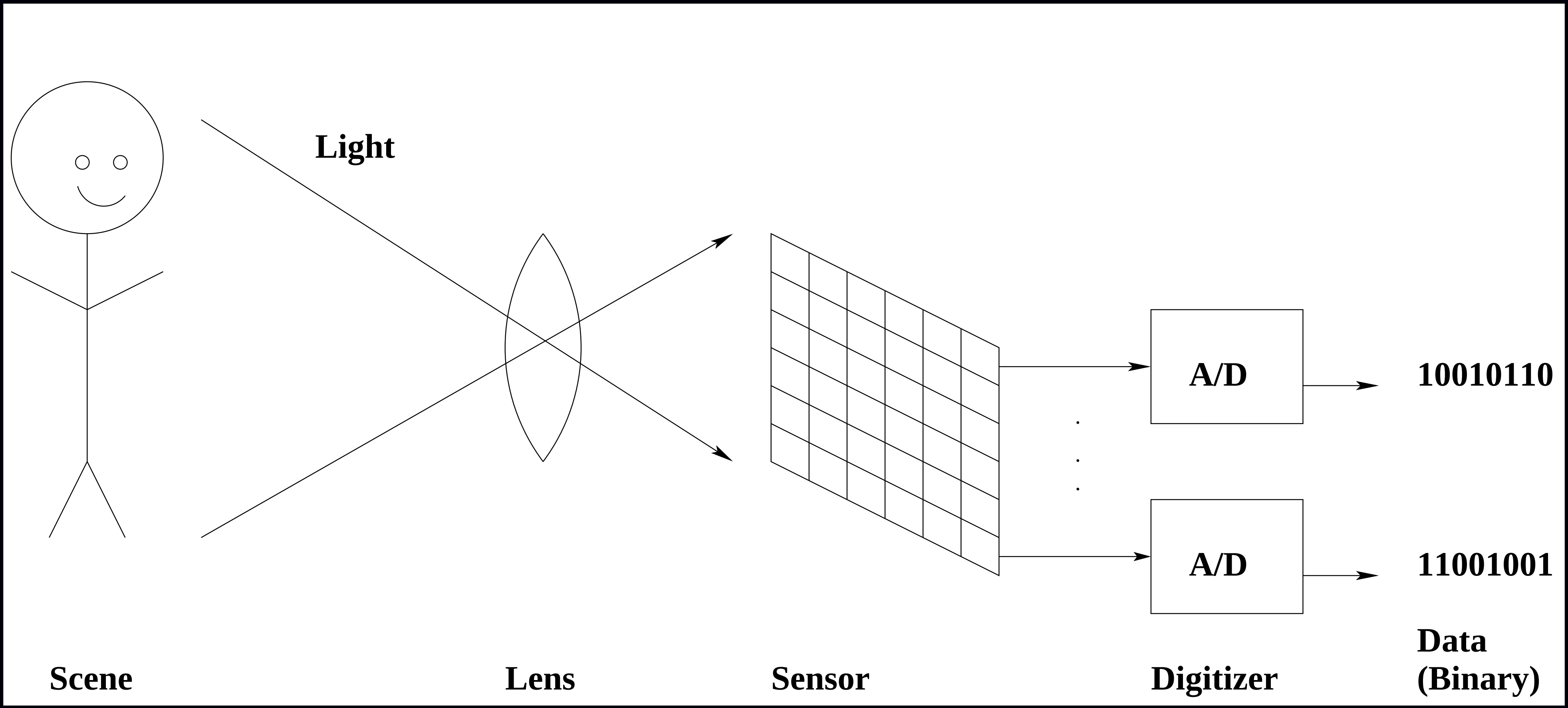
Cell Phones Everywhere

Notice anything funny about this picture?

What is Inside an iPod?

This talk concerns the invisible algorithms executing inside the chips, within iPods, digital cameras, cell phones, ...

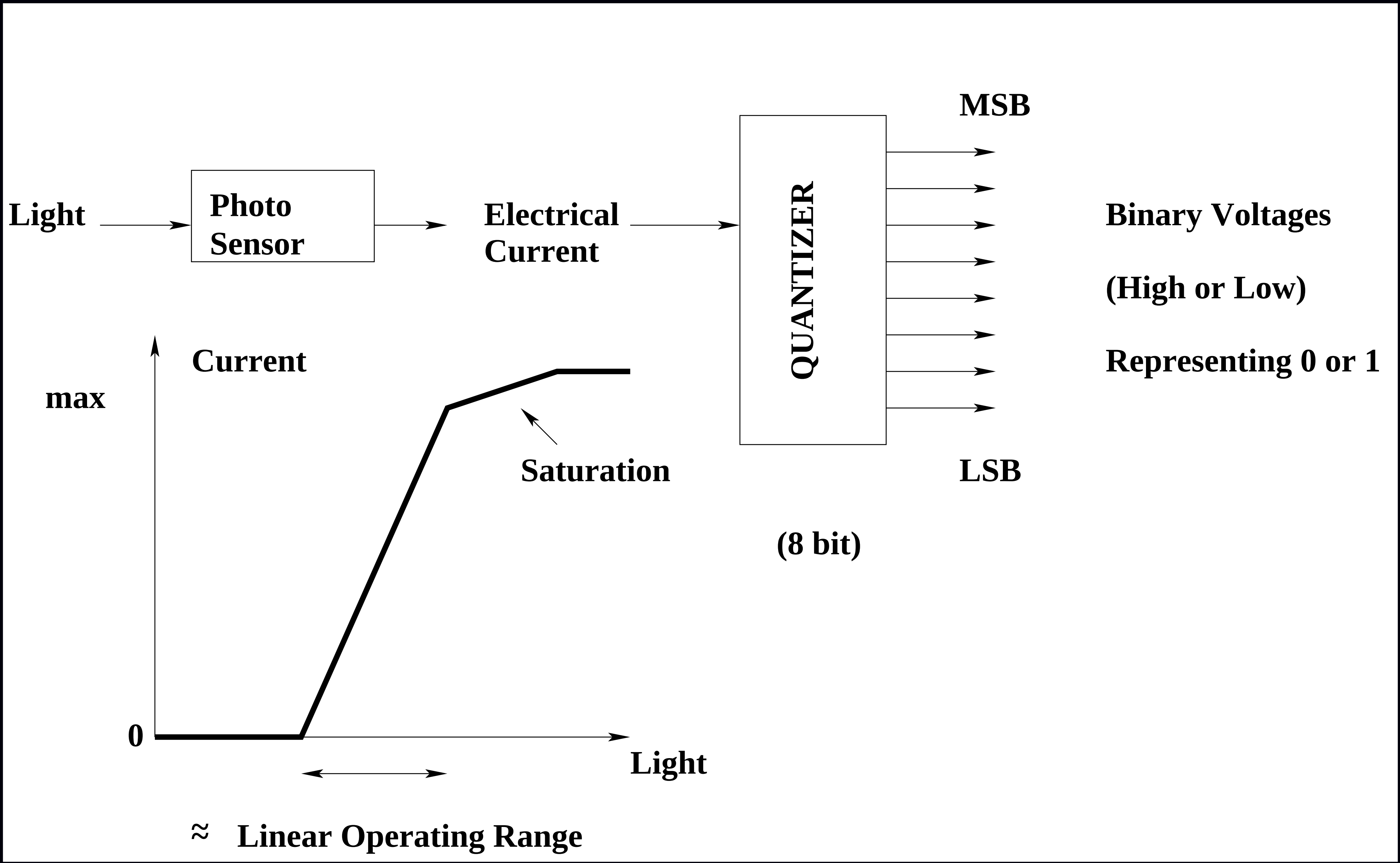
Digital Camera Basics



EECS everywhere: optics, semiconductor devices, analog circuits, digital logic, software, user interface ...

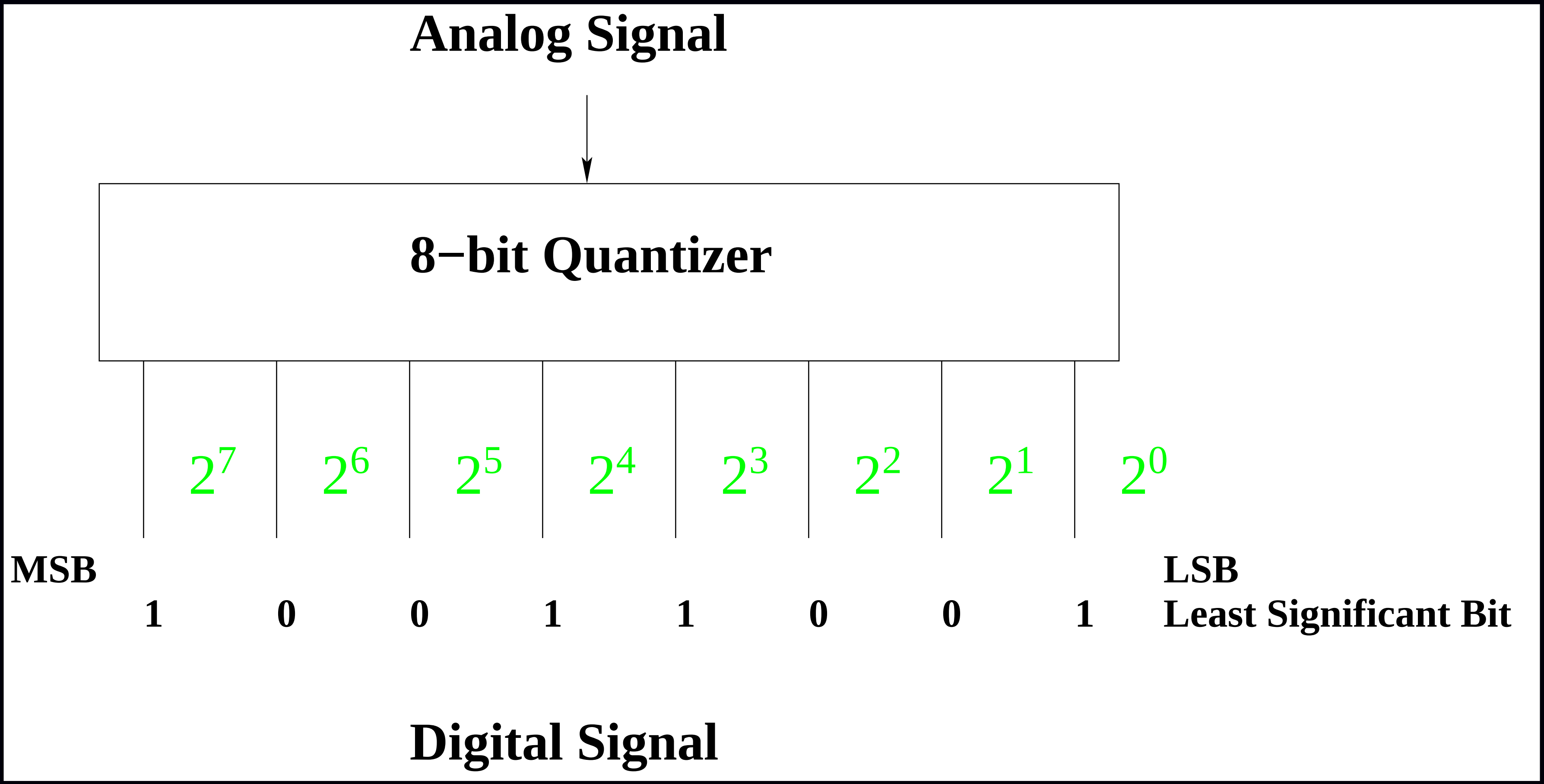
Key component: analog-to-digital converter

Analog-to-Digital (A/D) Conversion



Key component: quantizer

Quantizer



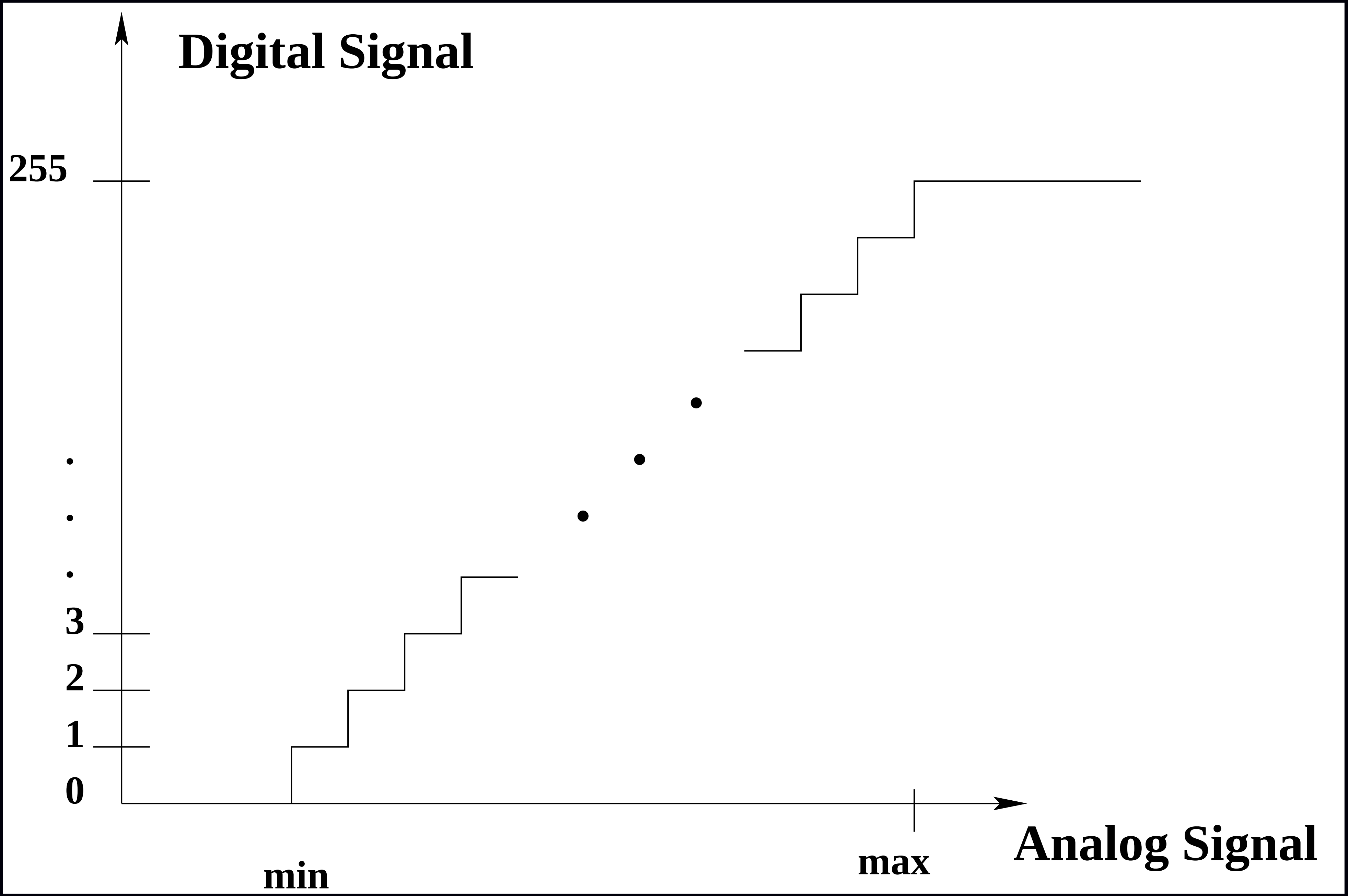
These 8-bit binary values represent:

$$2^7 + 2^4 + 2^3 + 2^0 = 128 + 16 + 8 + 1 = 153$$

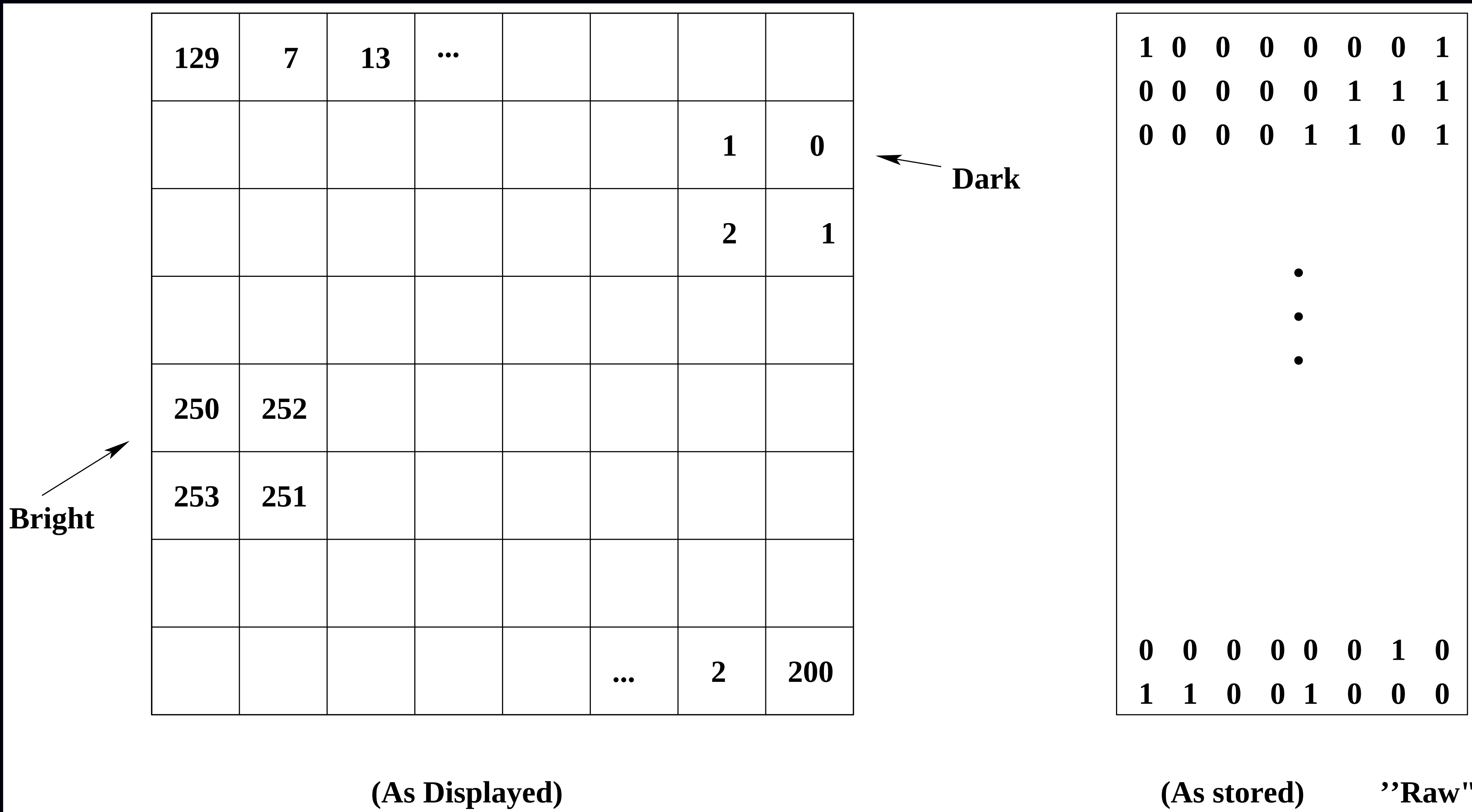
Smallest 8-bit digital value = 0.

Largest 8-bit digital value = ?

Quantization



Digital Gray scale (B/W) Image



$$\#\text{ bits stored} = \#\text{ pixels} \cdot \#\text{ bits per pixel (bpp)}$$

Lots of Bits...

Example: 3000×3000 pixel array (“9 Megapixel camera”)
8-bit quantizer

$$\begin{aligned} \text{total \# of bits} &= 3000 \cdot 3000 \cdot 8 \\ &= 72 \cdot 10^6 \text{ bits} \\ &\approx 9 \text{ MB} \end{aligned}$$

9MB for a grayscale image is undesirably large.
And for a RGB color image we would need $3\times$ more.

Image Compression aka Image Coding

A subfield within the field of *data compression* aka *source coding*.
Other data compression problems: audio compression (MP3), text compression (zip, ...)

Goal: reduce the # bits, while trying to preserve image quality.

- ○ Lossless image coding: perfectly reversible
- ○ Lossy image coding: recover an approximation

Why? For storage, transmission, ...

Concept:

$$\begin{array}{c}\text{original}\\\text{bits}\end{array} \rightarrow \boxed{\text{coder}} \rightarrow \begin{array}{c}\text{fewer}\\\text{bits}\end{array} \cdots \rightarrow \boxed{\text{decoder}} \rightarrow \begin{array}{c}\text{original}\\\text{bits}\end{array} \rightarrow \boxed{\text{user}}$$

The coder and decoder ("codec") are designed together.

Examples: MP3, JPEG, MPEG4, AVI, ...

Q: How low can we go (how few bits per pixel?)
A: Claude Shannon's information theory

Basic Image Compression by Rounding Down

Suppose for each pixel we discard the two least significant bits (LSBs), *i.e.*, we set them to 0.

MSB | 0/1 | 0/1 | 0/1 | 0/1 | 0/1 | 0/1 | 0 | 0 | LSB

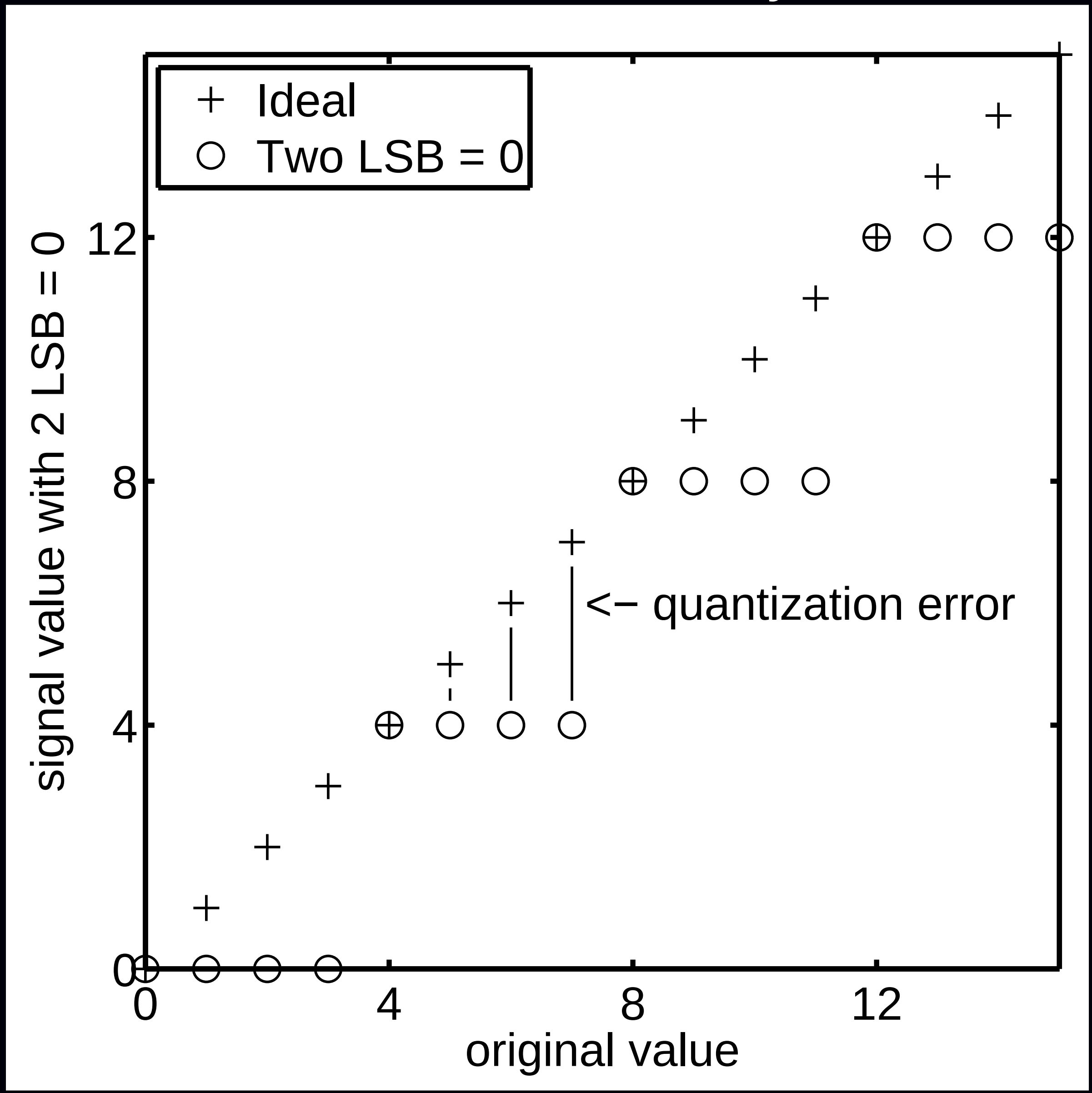
Possible gray scale values are multiples of 4:

00000000 = 0
00000100 = 4
00001000 = 8
00001100 = 12
00010000 = 16
⋮ ⋮
11111100 = 252

No need to store the two least-significant bits,
so now only 6 bits per pixel are stored, instead of 8.
25% data compression.
What happens to image quality?

Quantization Error for Rounding Down

Setting the two LSBs to 0 reduces memory, but induces error:



Average Quantization Error for Rounding Down

For discarding 2 bits:

$$\text{average error} = \frac{0+1+2+3}{4} = 1.5 \text{ gray levels}$$

For discarding d bits, where $0 \leq d < 8$:

$$\text{average error} = \frac{0+1+2+\cdots+2^{d-1}}{2^d} = \frac{2^d - 1}{2} \text{ gray levels}$$

As we discard more bits, the error increases.
Shannon called this the rate-distortion tradeoff.

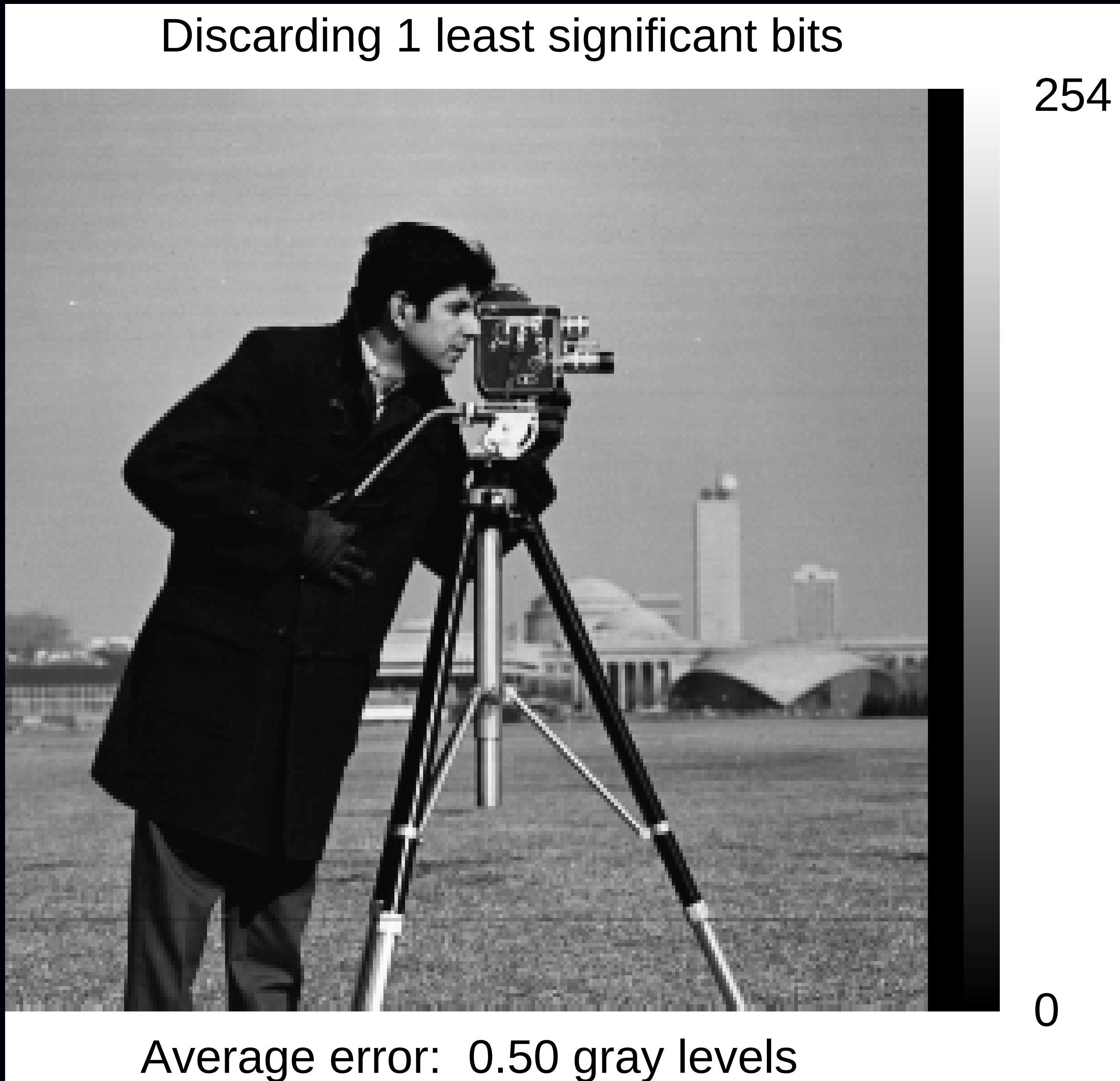
Next we see what it *looks* like.

Original Image

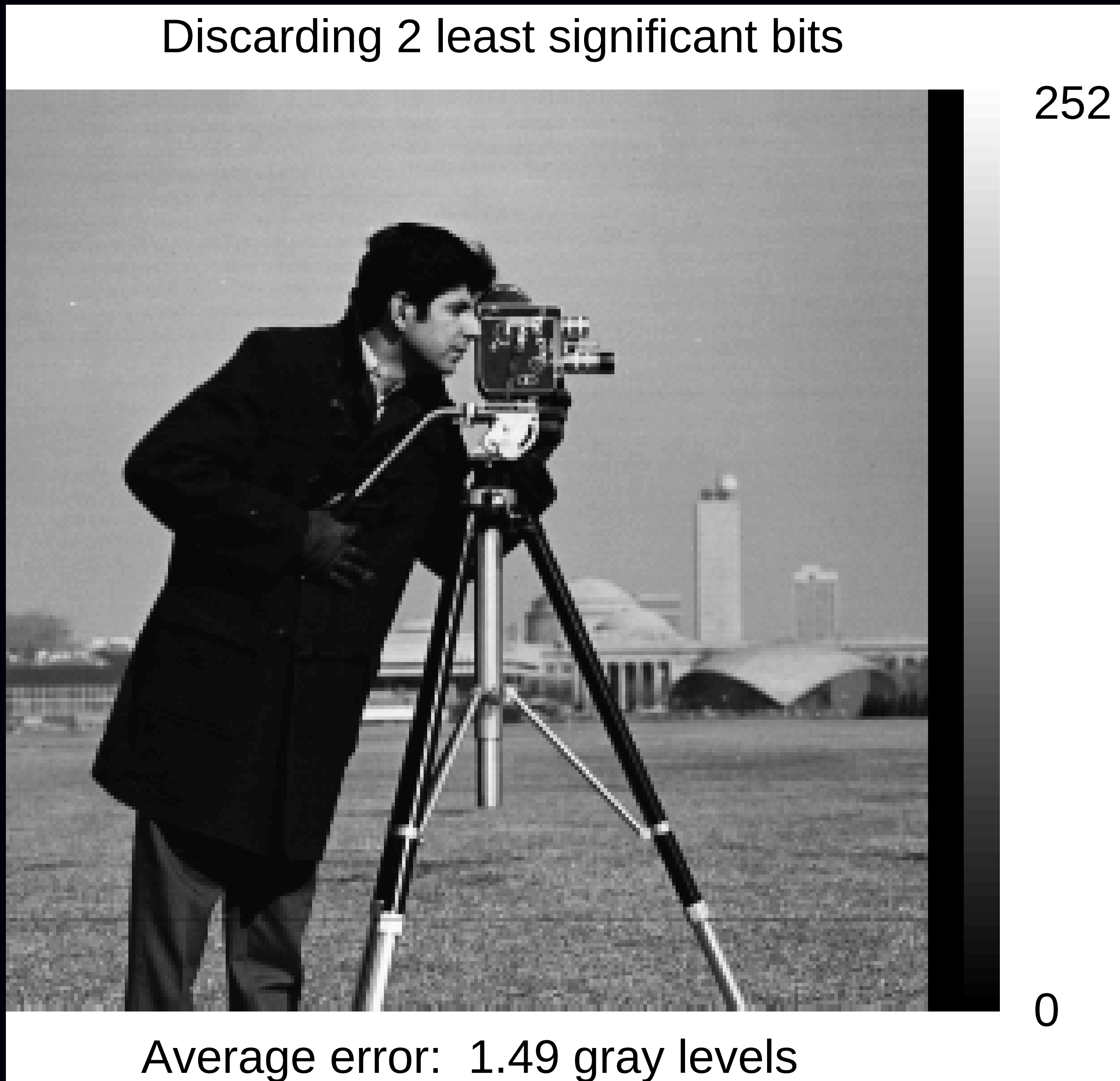
Original: using all 8 bits per pixel

Average error: 0.00 gray levels

Compressed Image 1



Compressed Image 2

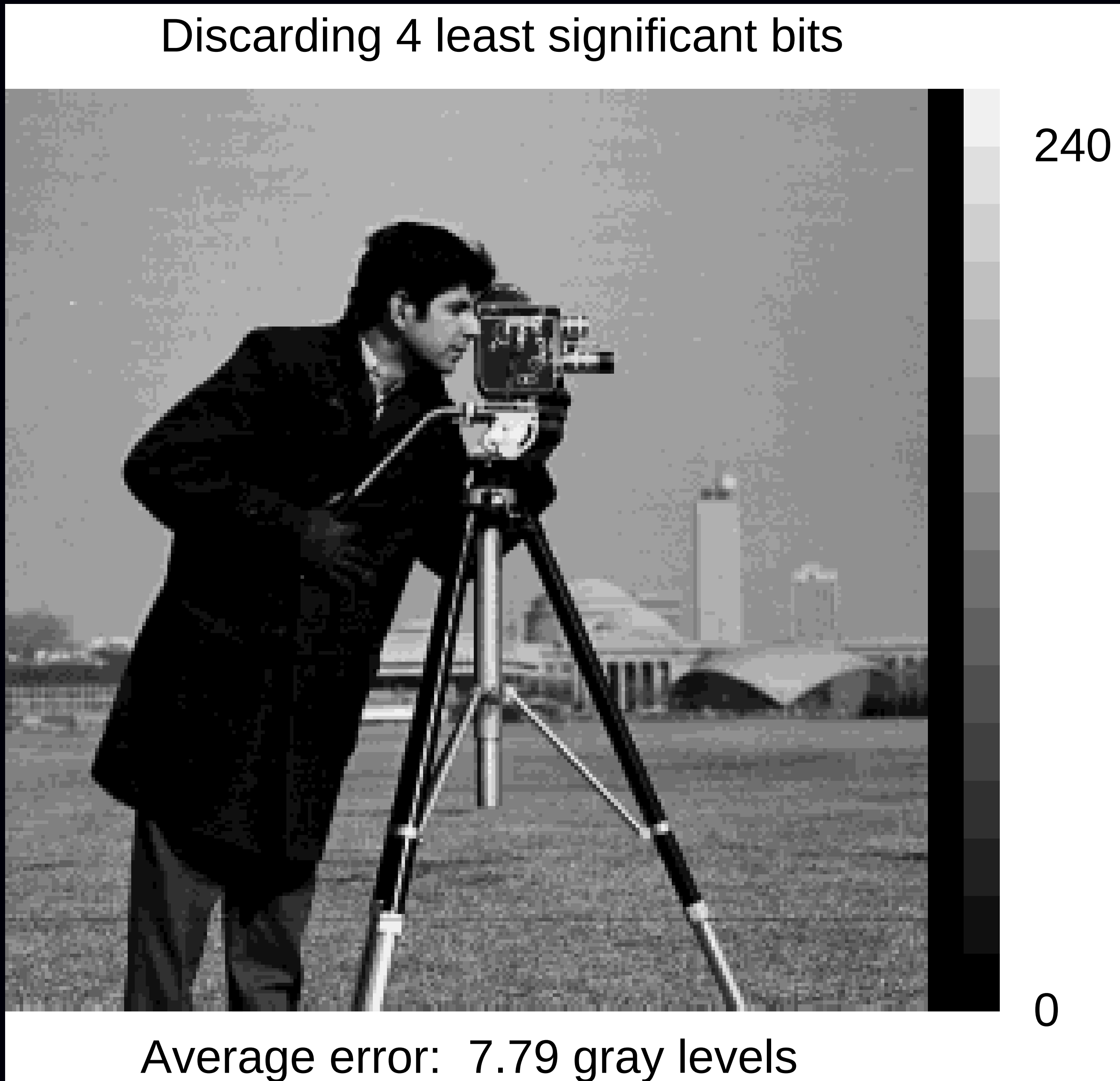


Compressed Image 3

Discarding 3 least significant bits

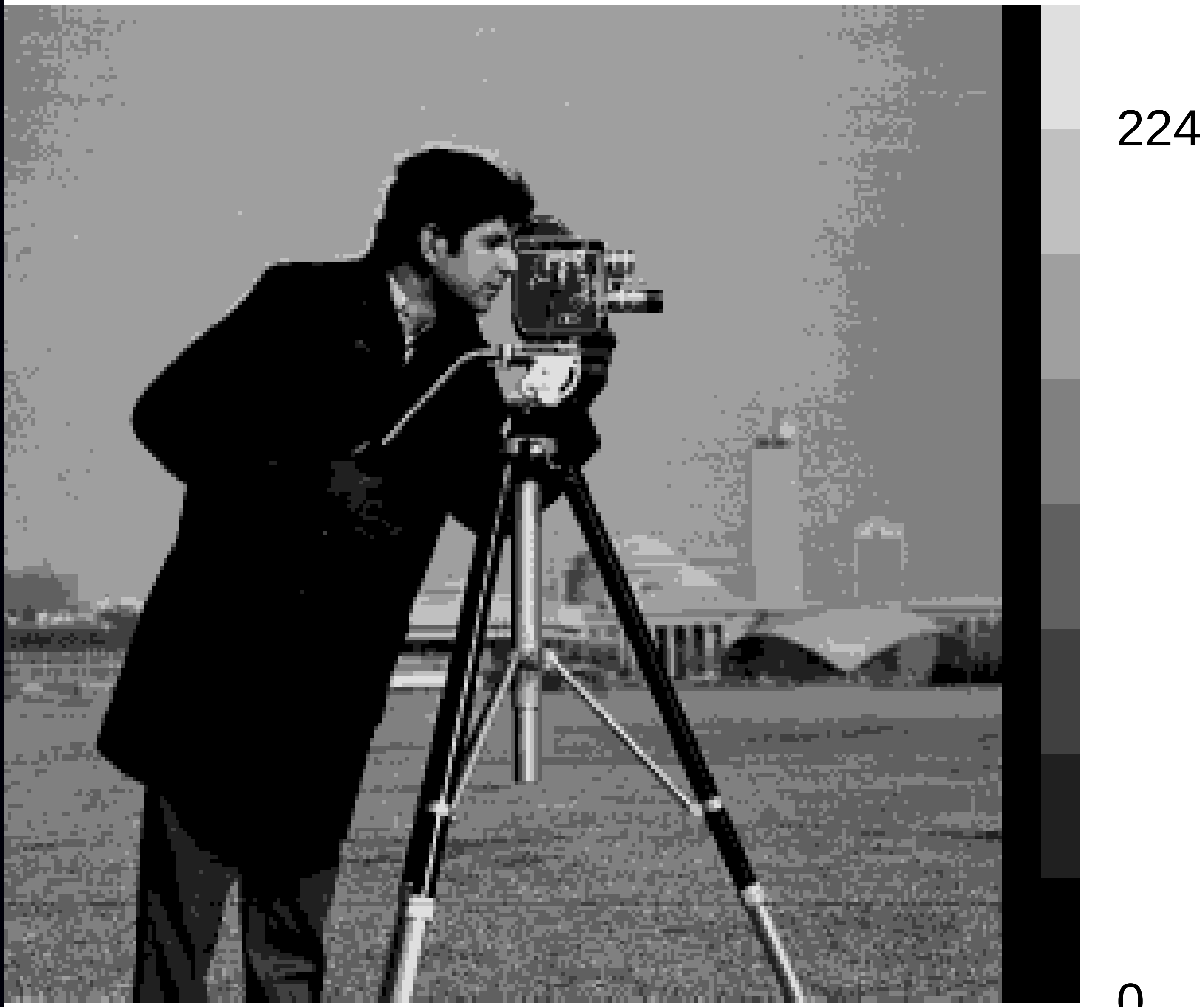
Average error: 3.48 gray levels

Compressed Image 4



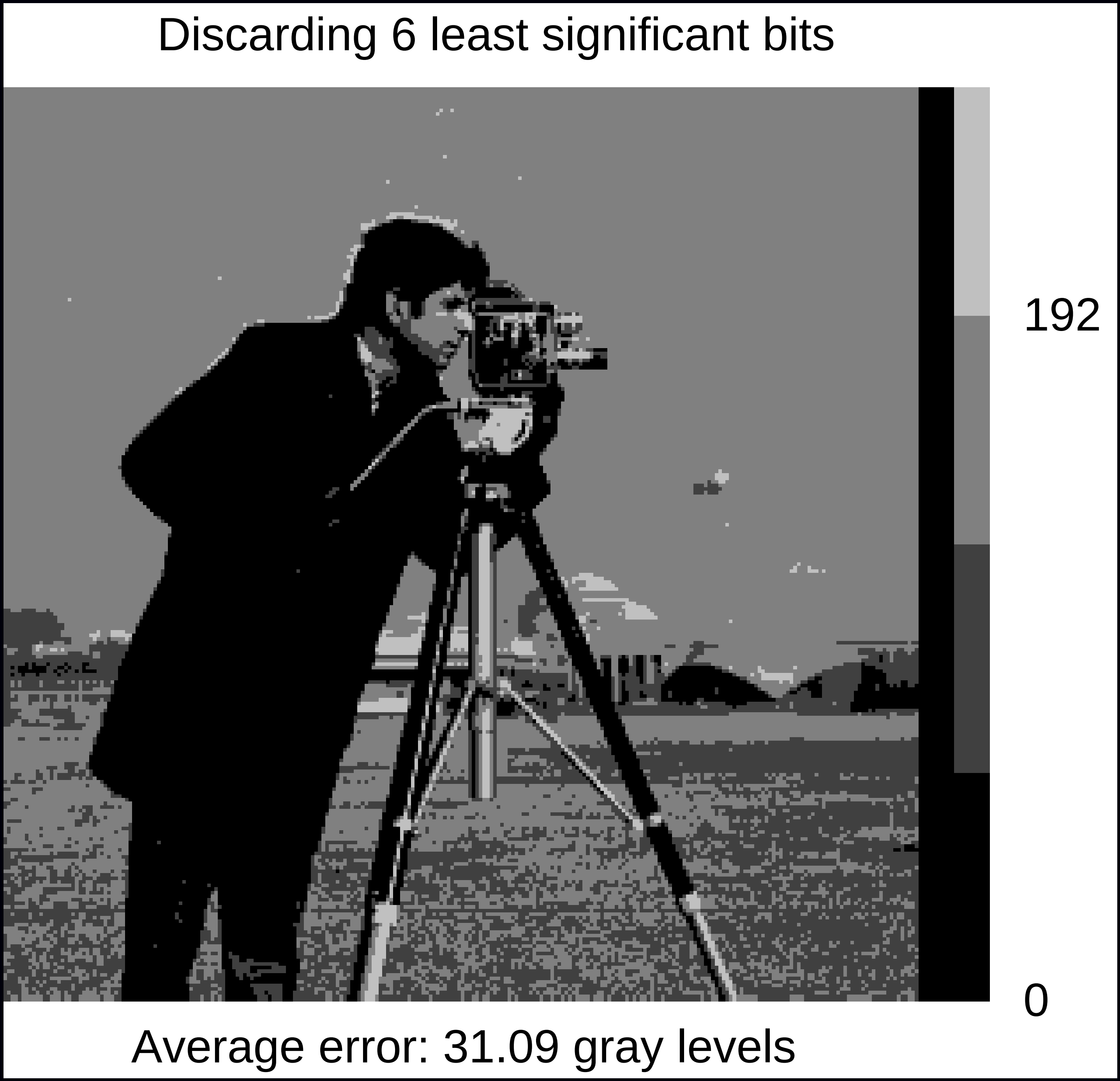
Compressed Image 5

Discarding 5 least significant bits

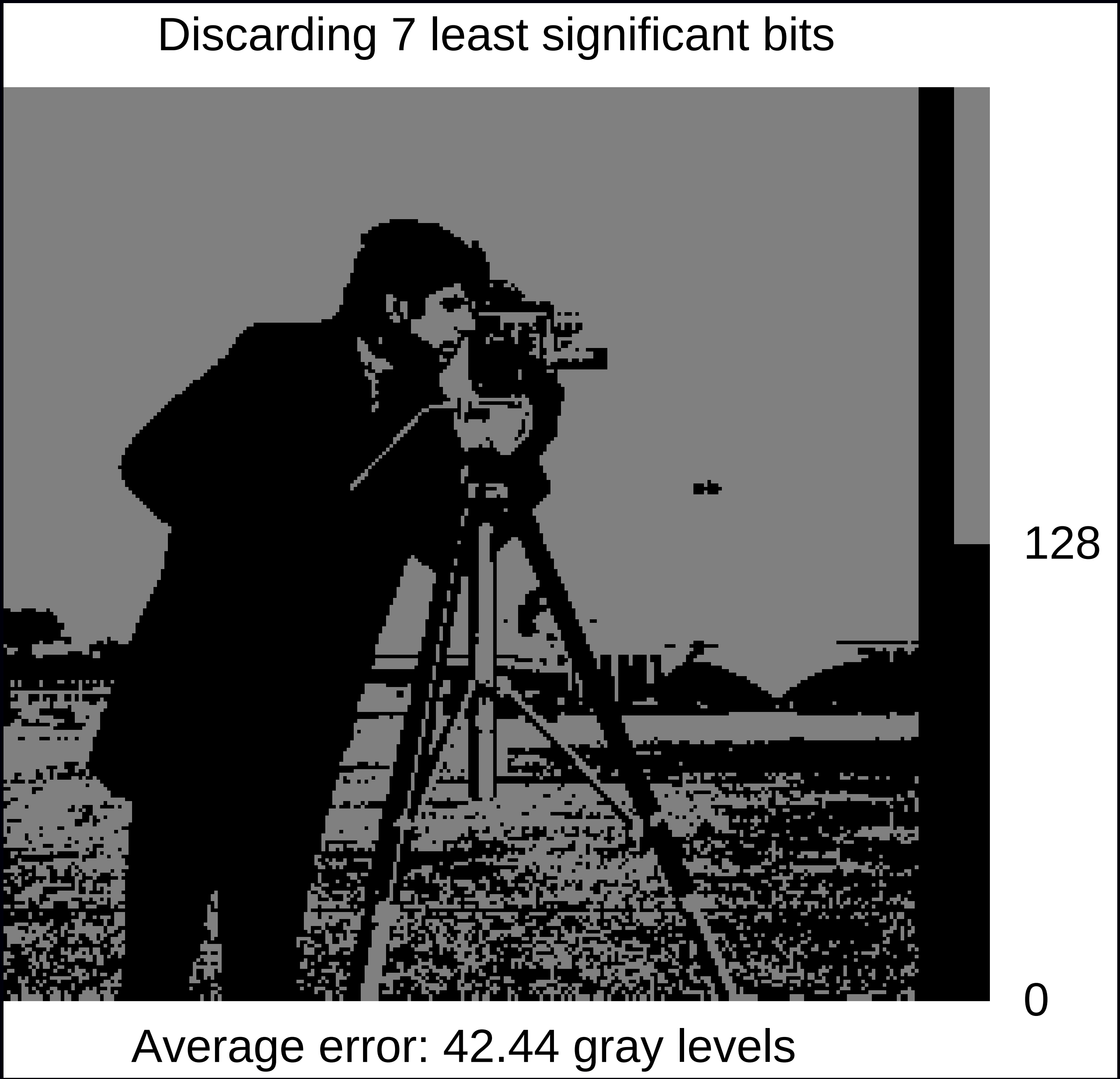


Average error: 14.61 gray levels

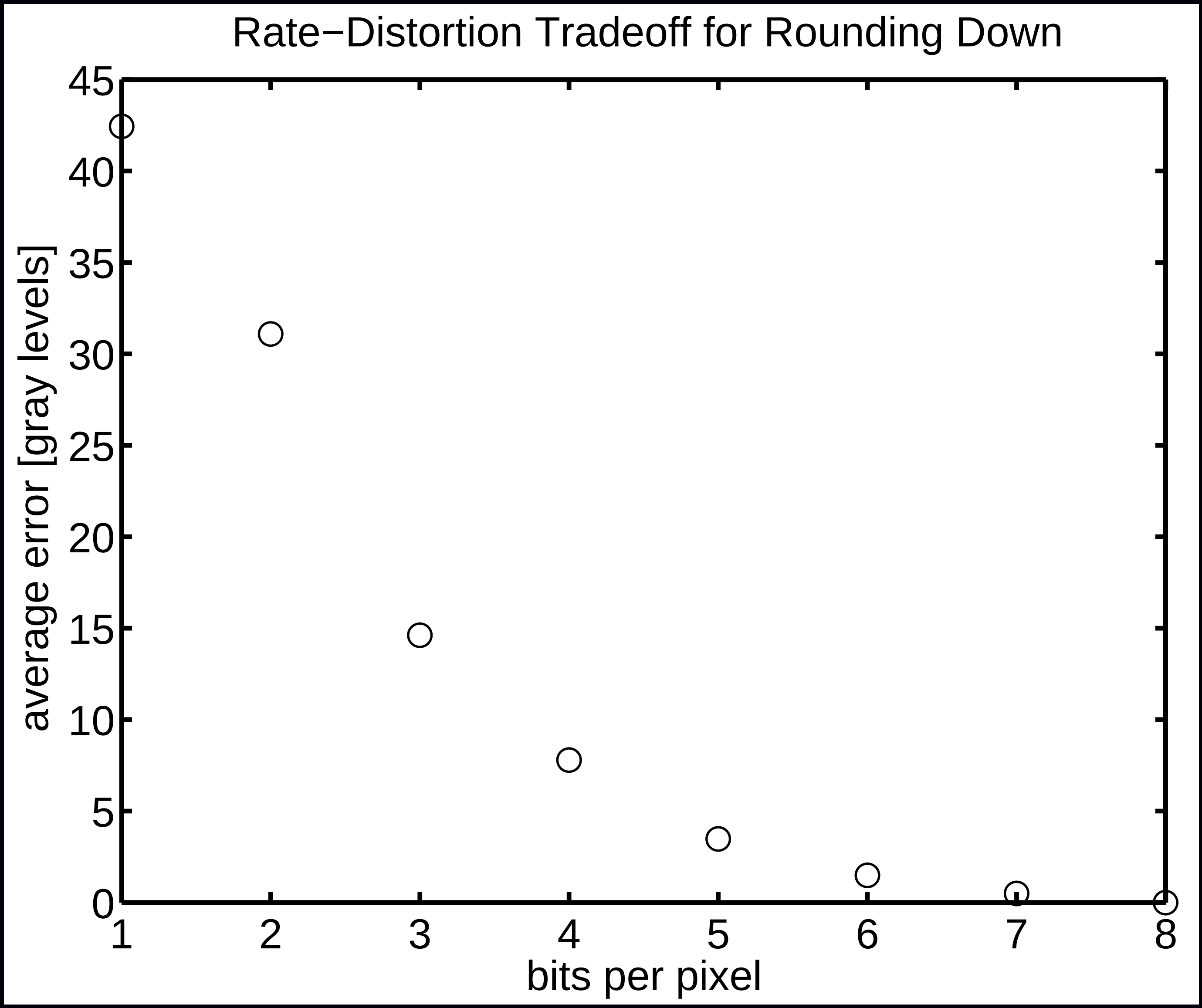
Compressed Image 6



Compressed Image 7

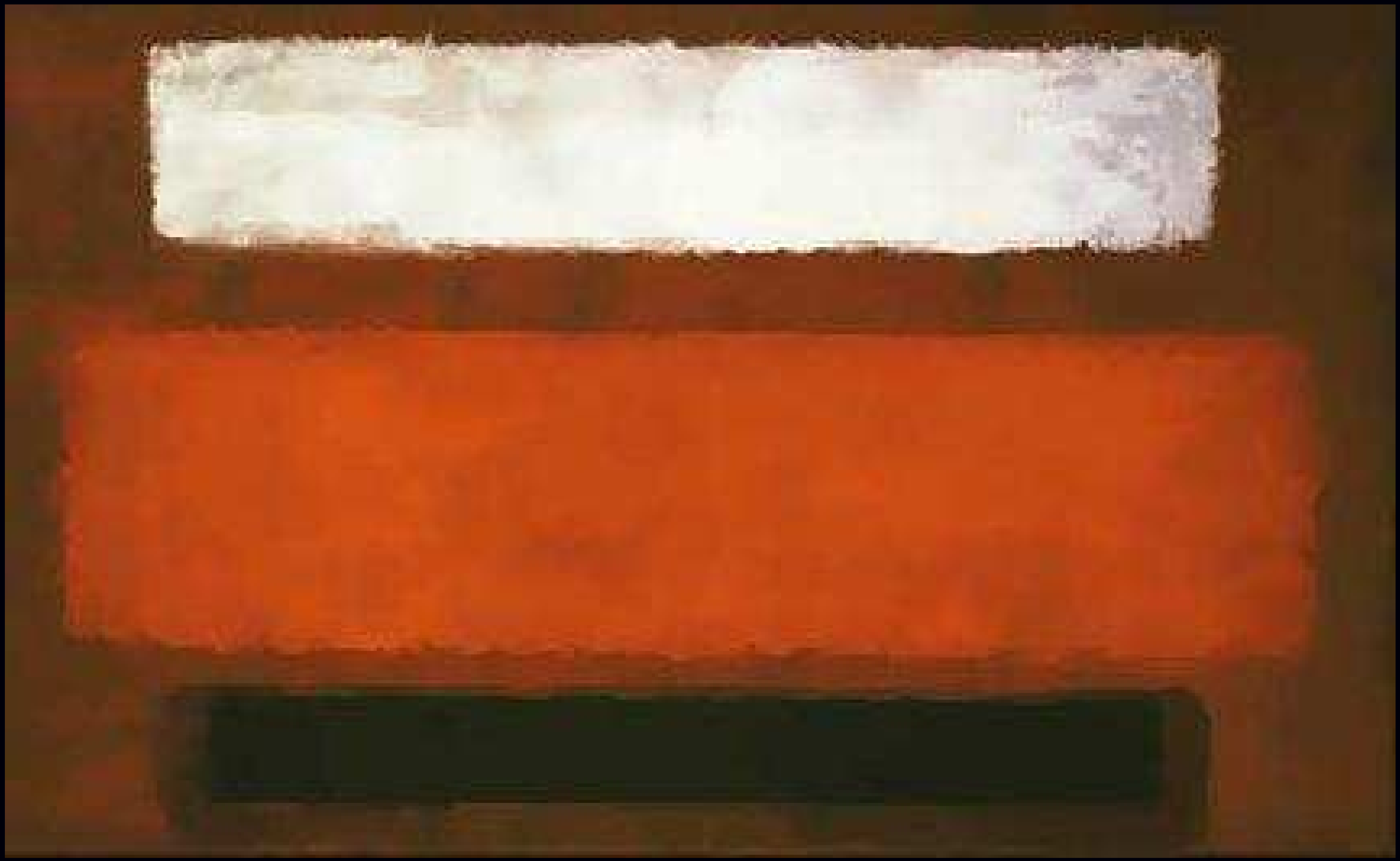


Rate-Distortion Tradeoff for Rounding Down



Can we design a better image compression method?
(What does better mean?)

Mark Rothko's "White and Black on Wine"



Jeff Fessler's "With Apologies to Rothko"

This image has only four distinct gray levels: 15, 120, 180, 230.
How many bits per pixel are needed to encode it?

Coding an image with a few distinct gray levels

Reasonable binary code:

value	code
15	00
120	01
180	10
230	11

With this code, only 2 bits per pixel are needed for this image.
(Plus a few bits overhead to store the code table for the decoder.)

Can we do even better?

Variable-Length Codes

So far we have been using fixed-length codes, where every value is represented by the same number of bits. (2 bits per value in preceding example.)

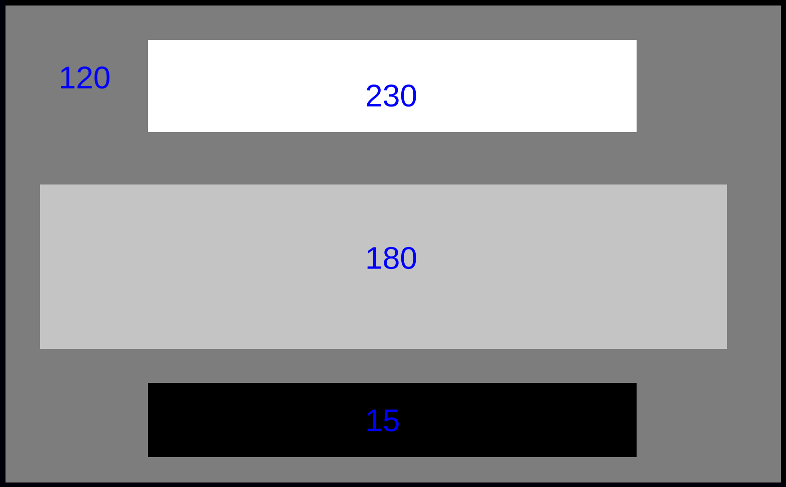
Consider (international) Morse Code (1840s):

A . -	G - - .	N - .	U . . -
B - . . .	H	O - - -	V . . . -
C - . - .	I . .	P . - - .	W . - -
D - . .	J . - - -	Q - - . -	X - . . -
E .	K - . -	R . - .	Y . - . -
F . . - .	L . - . .	S . . .	Z - - . .
	M - -	T -	

Why only a single “dot” for “E” ?

Idea of variable-length codes: use fewer bits for values that occur more frequently.

Variable-Length Code: Example



Improved (variable-length) code:

value	proportion	code
15	9.8 %	0 1 1
120	47.5 %	1
180	30.5 %	0 0
230	12.2 %	0 1 0

How many bits per pixel *on average* are needed with this code?

$$0.098 \cdot 3 + 0.475 \cdot 1 + 0.305 \cdot 2 + 0.122 \cdot 3 = 1.745$$

Less than 2 bits per pixel! *How is it stored?*

This is a Huffman code (see Matlab's `huffmandict` command).

Can we do even better?

Shannon's Source Coding Theory

To encode numerous signal values that lie in a set with N elements with proportions (probabilities) $p_1, p_2, \ldots, p_N$, on average we need *at least* H bits per value, where H is the entropy:

$$H = -\sum_{n=1}^{N} p_n \log_2 p_n.$$

Example: for our image with $N = 4$ gray levels, the entropy is:

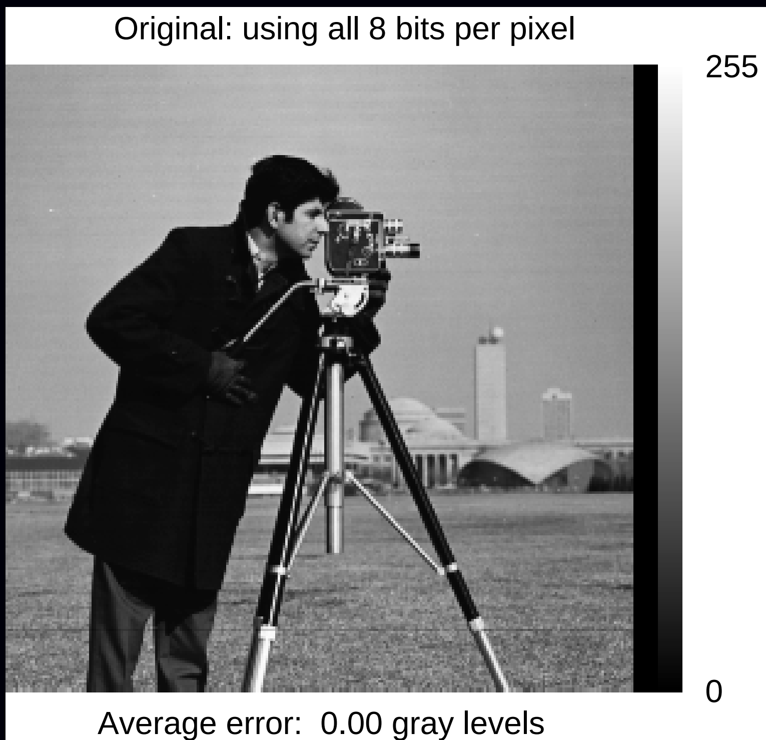
$$\begin{aligned} H &= -\left(p_1 \log_2 p_1 + \cdots + p_4 \log_2 p_4\right) \\ &= -\left(0.098 \log_2 0.098 + 0.475 \log_2 0.475 \right. \\ &\quad \left. + 0.305 \log_2 0.305 + 0.122 \log_2 0.122\right) \\ &\approx 1.7313 \text{ bits per pixel} \end{aligned}$$

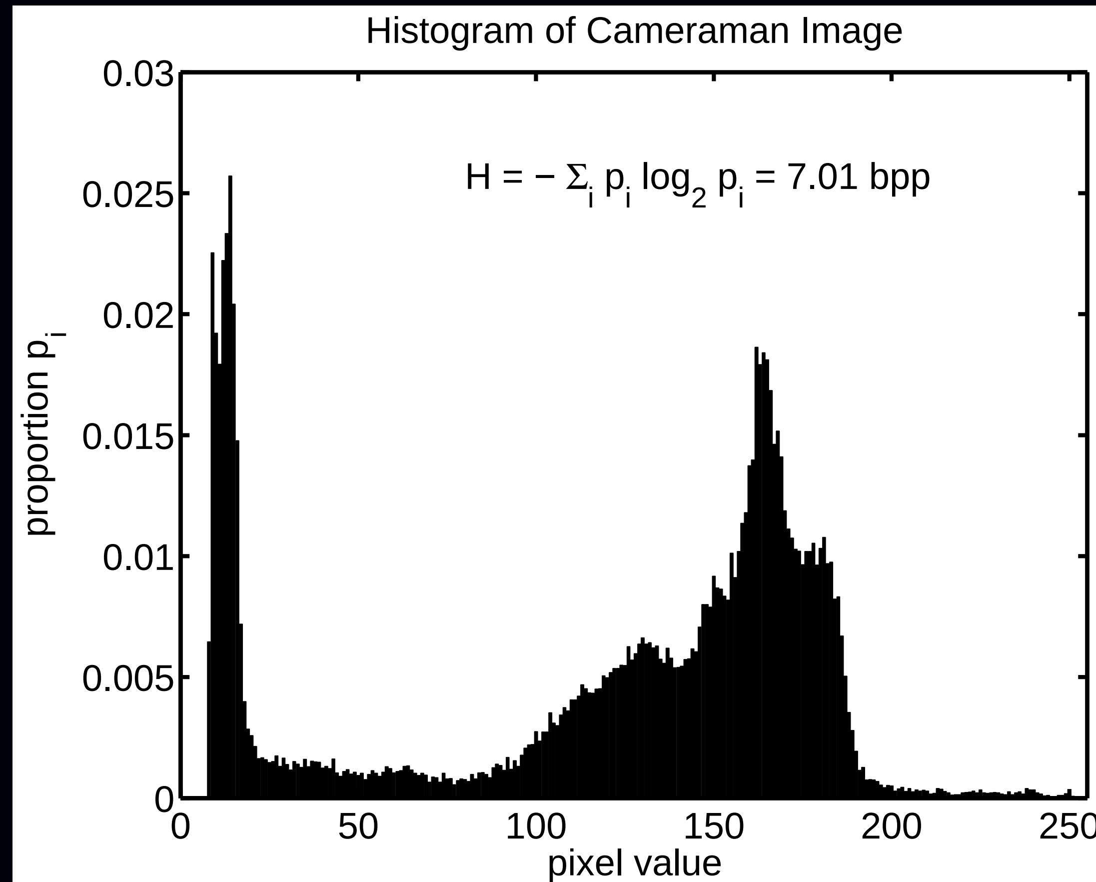
Our Huffman code came remarkably close to this lower limit.

This type of "theoretical bound" is important for *practical design*.

(note: analysis vs design)

More Complicated Images?





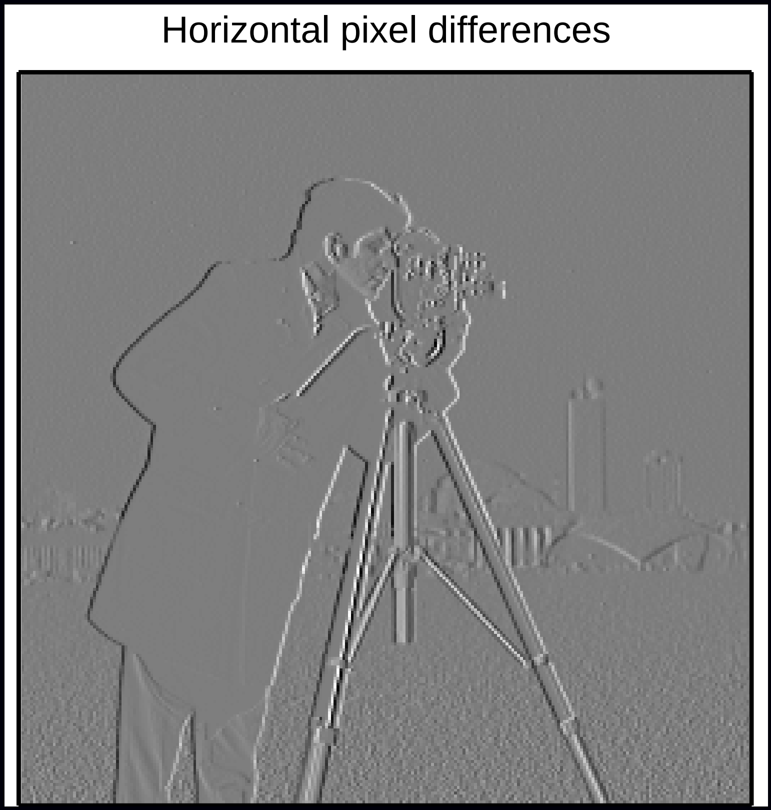
This image's pixel values also lie a finite set: $0, 1, 2, \ldots, 255$.
So Shannon's entropy bound applies.
For this image, need at least 7.01 bits per pixel.
Can we do better?

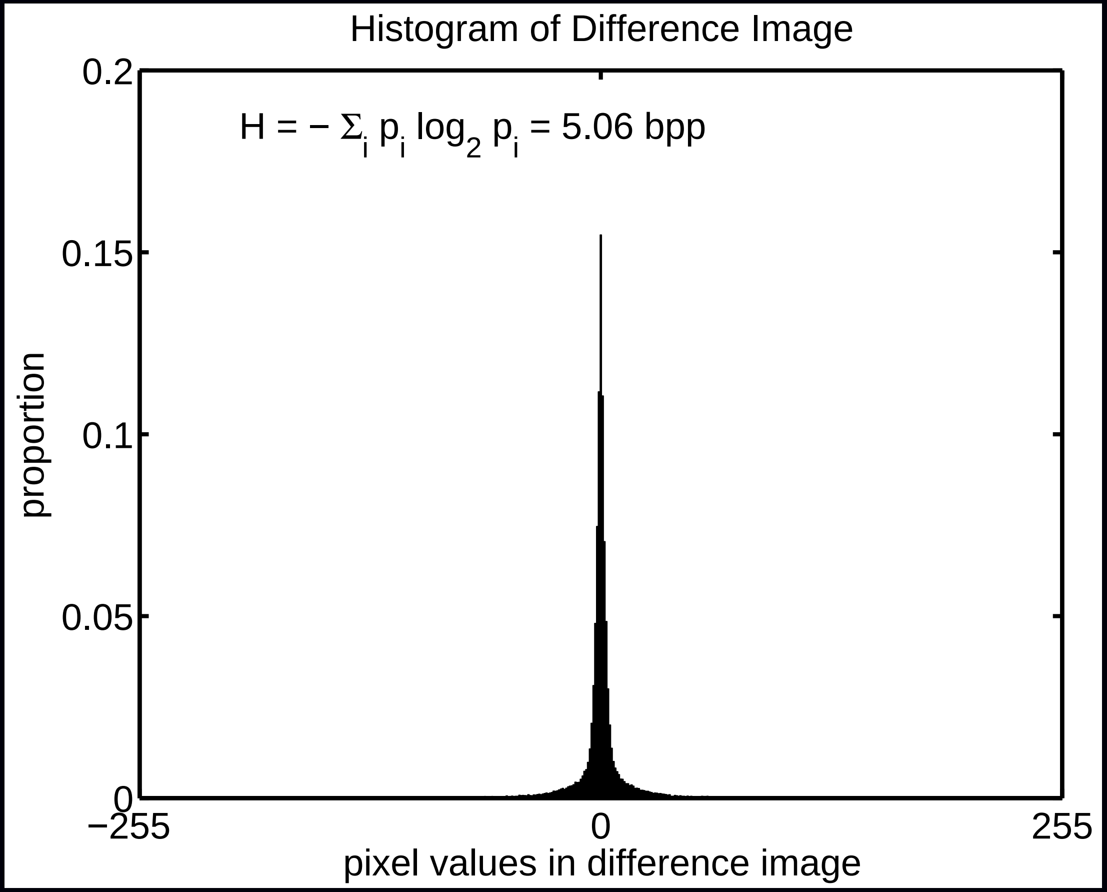
Coding Pixel Relationships

So far we have coded the *individual* pixel values directly.
This provides only modest data compression for most images.

For better image compression, we must consider the *relationships* between pixel values.
For example, neighboring pixel values tend to be similar.

Coding Pixel Differences





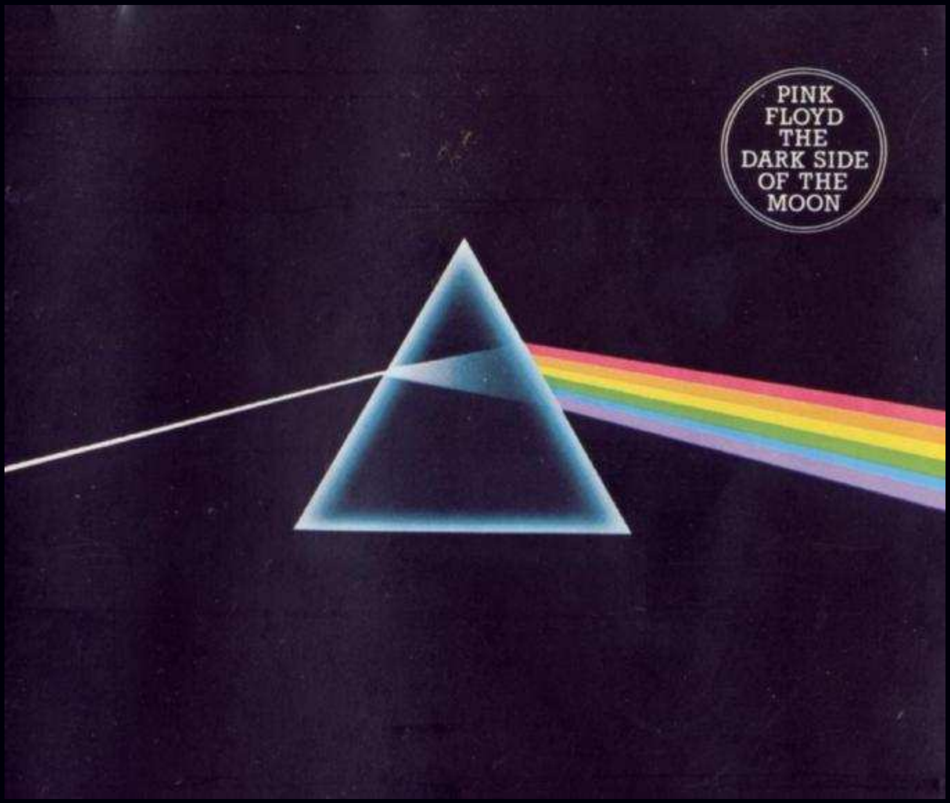
The horizontal difference image has pixel values that lie in a finite set: $-255, -254, \ldots, 0, \ldots, 255$.
So Shannon's entropy bound applies.
For this image, need at least 5.06 bits per pixel.

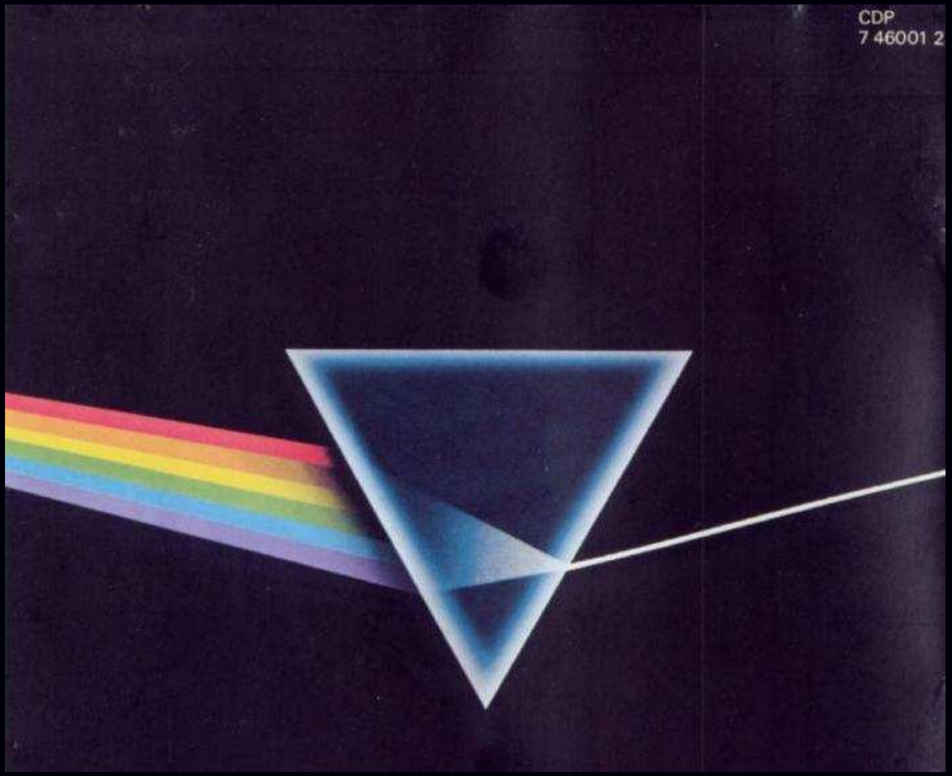
Modern Transform-Based Image Coding

- Capture pixel relationships using:
  - discrete Cosine transform (JPEG)
  - wavelet transform (JPEG 2000)
- Substantial compression by discarding small DCT coefficients
- Lossy vs lossless compression
- For video coding,
  - use DCT within each frame
  - and differences between frames

JPEG Image Compression and the DCT

DCT = discrete cosine transform
relative of the discrete Fourier transform (DFT)
practical thanks to the fast Fourier transform (FFT)
digital equivalent of an optical prism





Original Image

Original: using all 8 bits per pixel

JPEG Compressed Image 1: 100%

JPEG with 0.685 bits per pixel

Average error: 0.62 gray levels

JPEG Compressed Image 2: 95%

JPEG with 0.390 bits per pixel

Average error: 1.32 gray levels

JPEG Compressed Image 3: 80%

JPEG with 0.185 bits per pixel

Average error: 2.85 gray levels

JPEG Compressed Image 4: 25%

JPEG with 0.071 bits per pixel

Average error: 5.40 gray levels

JPEG Compressed Image 5: 5%

JPEG with 0.030 bits per pixel

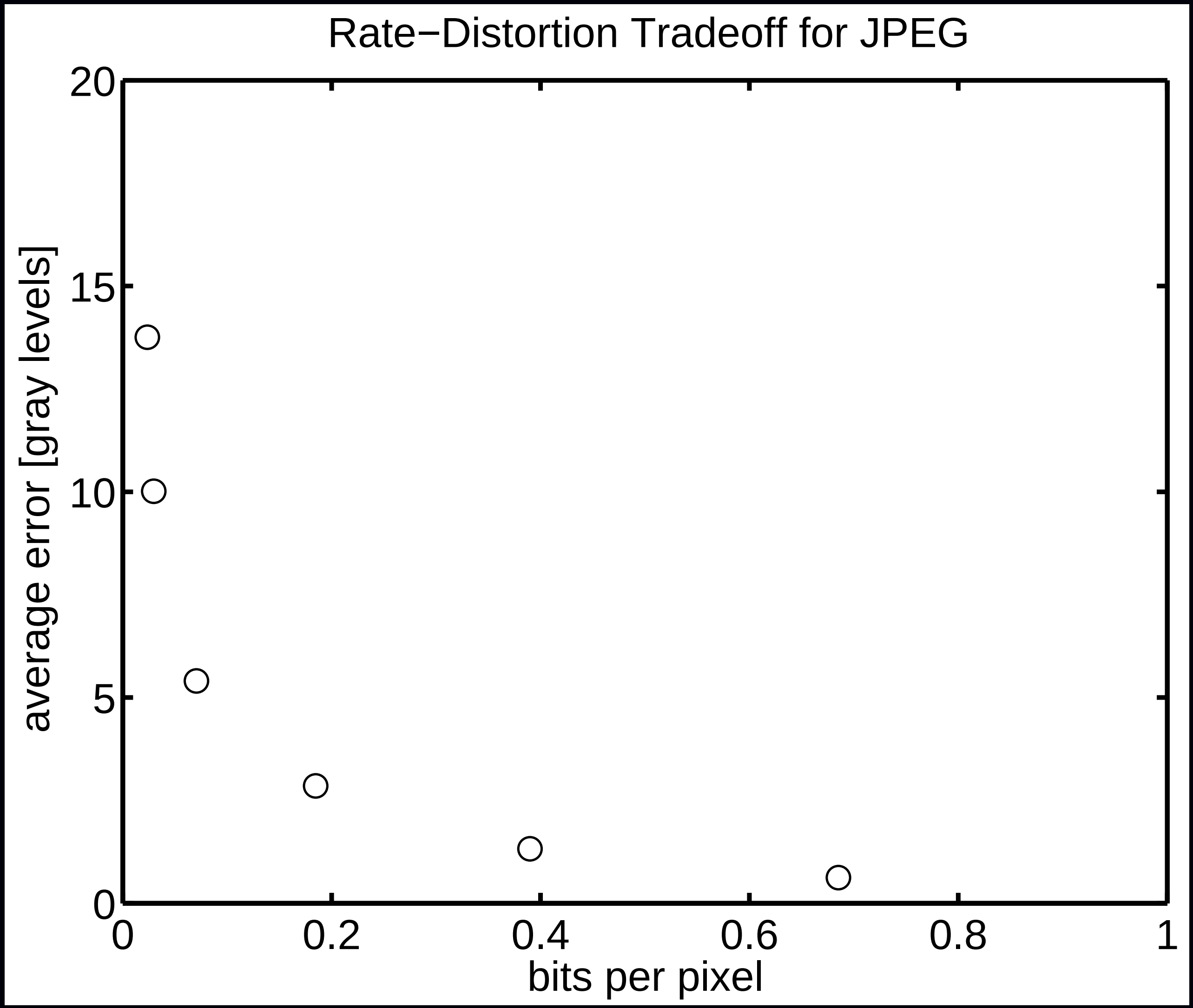
Average error: 10.01 gray levels

JPEG Compressed Image 6: 1%

JPEG with 0.024 bits per pixel

Average error: 13.76 gray levels

Rate-Distortion Tradeoff for JPEG



Recall that for compression by rounding,
at 1 bbp the average error was 40 gray levels.

Summary

- EEs are responsible for the principles that underly modern digital audio and video
  - both the devices / technologies (applied physics)
  - and the algorithms that they execute (applied math)
- Data compression is a very active research area
- Other considerations in data compression
  - complexity
  - hardware implementations
  - color
  - progressive encoding
  - ...
- To learn more about signals / systems / sampling / quantization:
  - EECS 216, 401, 451, 452, 455 (design / implement)
  - EECS 550, 551, 556, 650, 651, 750 (invent next generation)