Eng. 100: Music Signal Processing

DSP lecture

Final exam review

Announcements:

- Course evaluations: email receipt to `eecs-evals@umich.edu`
- Check scores on Canvas
- Final Exam: Thu. Dec. 17, 4-6 PM, 1500 EECS

Project 3 questions?

Final exam

- Final Exam: Thu. Dec. 17, 4-6 PM, 1500 EECS
  - not "Michigan time"
  - Draft DSP cover page on Canvas with formulas (per Exam 1)
  - Policies: no calculators or computers or other electronics, no notes or books. No paper or "blue books" needed.
- DSP topics: Labs 1-3, HW 1-4, Projects 1-3

Not on final exam

- "Missing frequencies" in scatter plots
- Calculus, complex numbers, Newtonian physics
- `Matlab function` or anonymous function `@`
- Harmonic power spectrum
- Phase vocoder, FM synthesis

Resources on Canvas for review

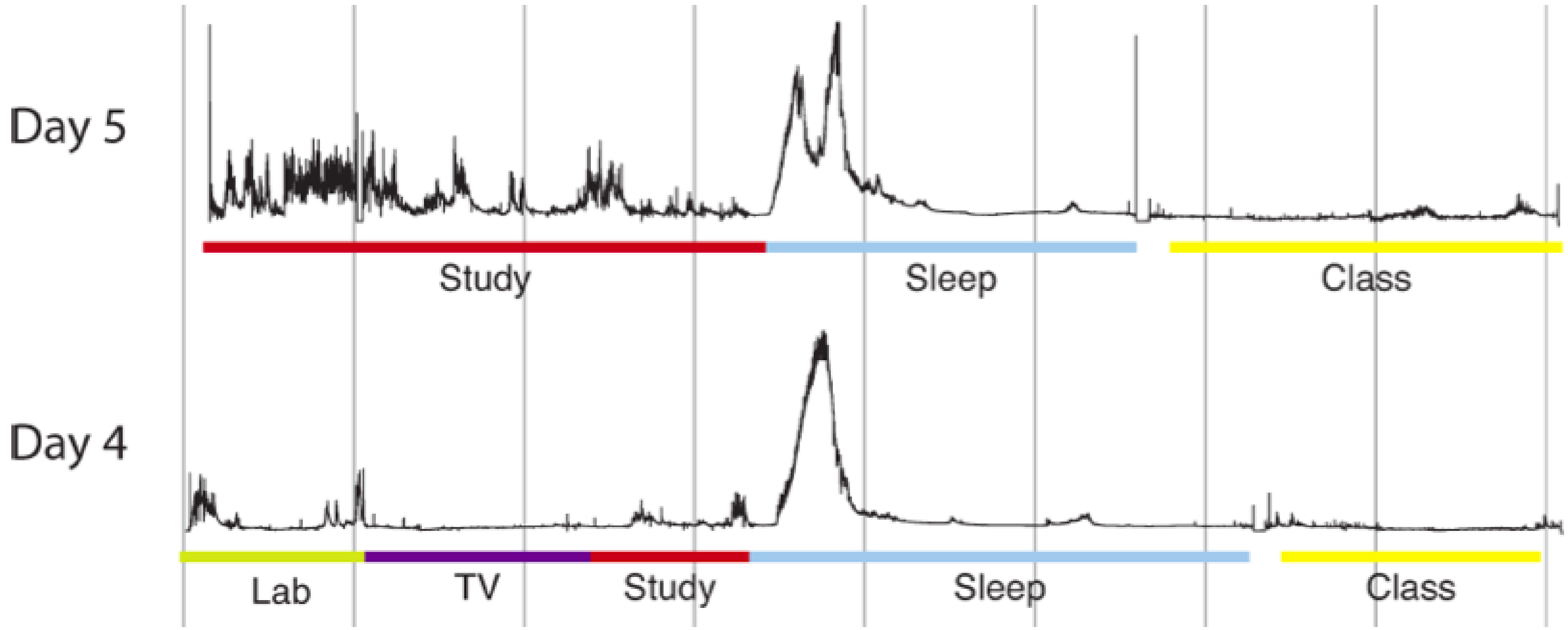
- Lecture slides
- Homework / solutions
- Prof. Yagle's course summary
- Prof. Yagle's tips on how to prepare for an exam.
- Review of spectra (Canvas: resources/lectures)
- Practice exam(s)
- Exam 1 / solutions
- Labs

  For each `Matlab` line that you "cut and paste" can you:
  - describe what the purpose of that command was?
  - determine the size of the output array?
  - determine number of points that are plotted?
  - determine what would be appropriate labels for the plot?
  - determine by hand what value would be computed?

Lecturing vs engaging

Ming-Zher Poh, M C Swenson, R M Picard; "A wearable sensor for unobtrusive, long-term assessment of electrodermal activity" IEEE Tr. on Biomedical Engineering, 57(5):1243-52, May 2010.

http://dx.doi.org/10.1109/TBME.2009.2038487



Spectrum review sheet here

Determining the frequency of a sinusoid

- numerous applications, not just music
- Analog methods
  - From continuous-time formula: by inspection $x(t) = 200\cos(2\pi 100t + 30)$
  - From plot: find (fundamental) period T, then use $f = 1/T$
  - From sound: "tuning" like musicians do using beat frequencies
  - From line spectrum: by inspection
  - From a text description:
    "the 2nd harmonic of a periodic signal having period of 10 msec" ??
- Digital methods "by hand"
  - From formula: $x[n] = 7\cos\left(\frac{\pi}{8}n\right)$ where $S = 8000$ Hz
    ??
  - From plot?
  - From `Matlab` code: `S = 100; x = cos(2*pi*[1:50]*40/S);`
  - ...

Digital methods

- arccos method (non-redundant conditions for it to work?)
- FFT: peak in spectrum (at "$k+1$") then $f = \frac{k}{N}S$
- correlation:
  - correlate with (numerous?) candidate sinusoids
  - use both cos and sin
  - choose frequency associated with maximum correlation
- autocorrelation:
  - correlate signal with shifted versions of itself.
  - ignore peak at 0; find 2nd peak (at "$m+1$") then $f = S/m$
- Harmonic power spectrum?
- Spectrogram (many FFTs, again using $f = \frac{k}{N}S$)

Sampling

- B (for "bandwidth") denotes highest frequency of a signal
- What is the largest value B can have? ??
- Example of a signal that "achieves" that upper limit? ??
- To avoid aliasing, use $S > 2B$
- All of the spectrum-related methods require $S > 2B$ to work

Example. Suppose the signal $x(t) = 7\cos(2\pi 250 t)$ is sampled with $S = 400\frac{\text{Sample}}{\text{Second}}$ for $N = 800$ samples.
Sketch the figure produced by the `Matlab` command
`stem(2/N*abs(fft(x)))`

What frequency would be found by the FFT method? ??

This example involves sampling, aliasing, spectra, and FFT.

Aliasing explanation for the example

$x(t) = 7\cos(2\pi 250t)$ when sampled at $S = 400\text{Hz}$ yields

$$x[n] = x(t)\Big|_{t=n/S} = 7\cos\left(2\pi 250\frac{n}{400}\right) = 7\cos\left(2\pi\frac{250}{400}n\right), \quad n \in \mathbb{Z}$$

$y(t) = 7\cos(2\pi 150t)$ when sampled at $S = 400\text{Hz}$ yields

$$y[n] = y(t)\Big|_{t=n/S} = 7\cos\left(2\pi 150\frac{n}{400}\right) = 7\cos\left(2\pi\frac{150}{400}n\right), \quad n \in \mathbb{Z}$$

But for all $n \in \mathbb{Z}$ (integers):

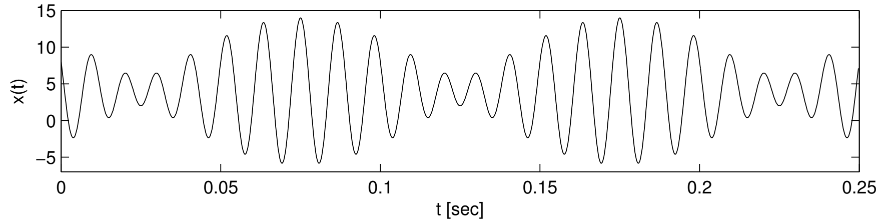
$$\cos\left(2\pi\frac{250}{400}n\right) = \cos\left(-2\pi\frac{250}{400}n\right) = \cos\left(-2\pi\frac{250}{400}n + 2\pi n\right) = \cos\left(2\pi\frac{150}{400}n\right)$$

Where did we use the fact that n is an integer? ??

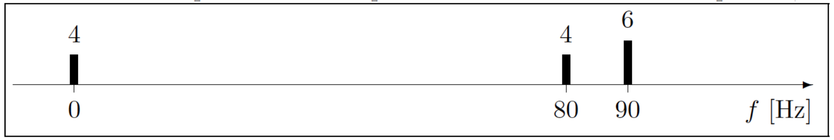
So even though $x(t)$ and $y(t)$ are completely different signals, after sampling $x[n]$ and $y[n]$ are the same. This is aliasing.

Signal representations review

- A *plot* of signal as a function of time: $x(t)$ versus t, such as



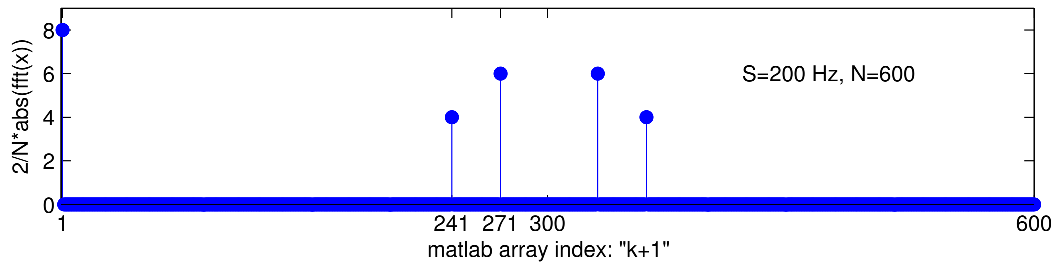
- A *"complicated" formula*, such as $x(t) = 8\cos^2(2\pi 40t) - 6\sin(2\pi 90t)$
- A *simple formula* as a sum of sinusoidal signals (cosines), such as (using trigonometric identities): $x(t) = 4 + 4\cos(2\pi 80t) + 6\cos(2\pi 90t + \pi/2)$.
- A *list* of the frequency, (positive) amplitude, and phase (f_k, c_k, θ_k) of each component, such as $(0, 4, \text{N/A})$; $(80, 4, 0)$; $(90, 6, \pi/2)$.
- A *line spectrum* that shows the frequencies and amplitudes of each sinusoidal component, such as:



- *Matlab commands*, such as: `t = [0:999]/8192;`

```
x = 4 + 4*cos(2*pi*80*t) + 6*cos(2*pi*90*t+pi/2);
```

- The *FFT output*, *i.e.*, the output of the Matlab command
  `2/N*abs(fft(x))`
  such as:



  Note that this Matlab plot differs somewhat from the line spectrum that we draw by hand.
  - The first array value is $2c_0$, so it is "twice as big as it should be."
  - The horizontal axis is not frequency f in Hz.
    (We could fix that by using another argument to `stem` command.)
  - Each of the nonzero lines appears twice (mirror image).
    How could we fix that? ??

Review how to convert between each of these representations!

Spectrum review problem

Example. Sketch the spectrum of the audio signal that is produced by the following `Matlab` code.

```
S = 44100; N = 0.5 * S;
t = [0:N-1]/S;
x = [2:3:8] * cos(2 * pi * ([3:5] * 200)' * t);
sound(x, S)
```

Hint. First think about the sizes of all arrays used above.