Eng. 100: Music Signal Processing

DSP Lecture 6 (addendum)

Aliasing

2015-10-13

- Needed for HW 3 !

Sampling rates and maximum frequency

- What is the highest frequency we could find by arccos method? (HW1 "challenge" problem.)
  The largest value of arccos is π when its argument is -1, so
  $f \le \frac{S}{2\pi}\arccos(-1) = \frac{S}{2\pi}\pi = \frac{S}{2}$, *i.e.*, $f_{\max} \le S/2$

- What is the highest frequency we can find by the FFT method?
  `disp((2/N)*abs(fft(x)))` gives:
  $[2c_0 \; \underbrace{c_1 \; c_2 \; \ldots \; c_{N/2-2} \; c_{N/2-1}} \; c_{N/2} \; \underbrace{c_{N/2-1} \; c_{N/2-2} \; \ldots \; c_2 \; c_1}]$

  The k for the highest frequency is $k = N/2$, so $f = \frac{k}{N}S \le \frac{N/2}{N}S = S/2$,
  i.e., $f_{\max} \le S/2$

- What is the maximum frequency we can find by eye from a digital signal $x[n]$?
  $T \ge 2/S$ so $f = 1/T \le S/2$

- What is the maximum frequency we can find by eye from an analog signal $x(t) = x(t+T)$? It can be arbitrarily high in principle.

Why $S > 2B$ is crucial to avoid aliasing

- Consider $x(t) = \cos(2\pi f t)$ with $f = S/2$
  Plot its samples $x[n]$
  $x[n] = x(n/S) = \cos(2\pi(S/2)(n/S)) = \cos(\pi n) = (-1)^n$

- Consider $y(t) = \sin(2\pi f t)$ with $f = S/2$
  Plot its samples $y[n]$
  $y[n] = y(n/S) = \sin(2\pi(S/2)(n/S)) = \sin(\pi n) = 0$

- Would $S \geq 2B$ suffice to avoid aliasing?
  No, because $\sin(2\pi f t)$ with $f = S/2$ has all zero samples so we cannot recover its frequency. We need $S > 2B$.

- In fact, for FFT, the highest frequency is really for $k = N/2 - 1$, *i.e.*, $f = \frac{(N/2-1)}{N} S = \left(\frac{1}{2} - \frac{1}{N}\right) S < S/2$

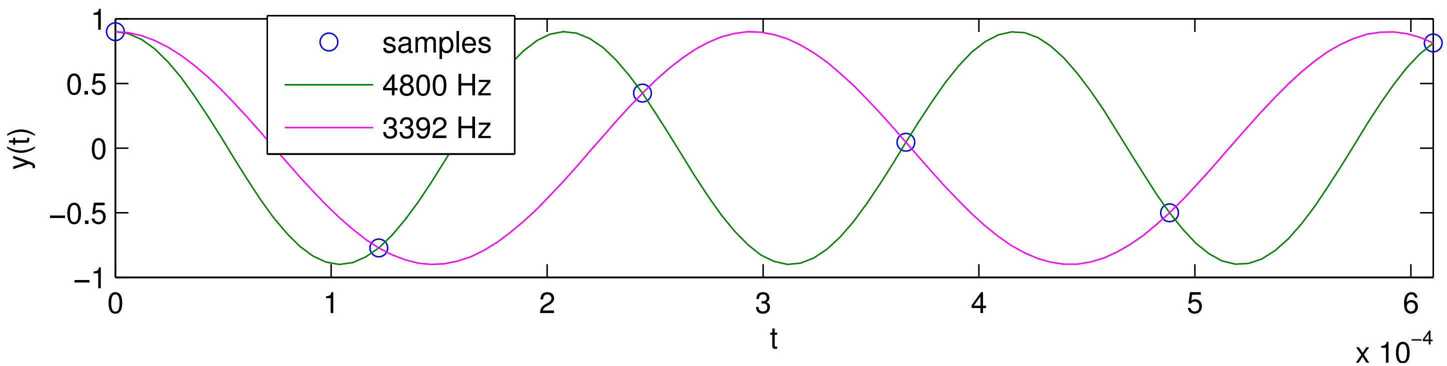
Aliasing: audio example

```
S = 8192; t = [0:1/S:0.3];
x = 0.9*[cos(2*pi*2800*t), cos(2*pi*3800*t)];
```

play

```
y = 0.9*[cos(2*pi*3800*t), cos(2*pi*4800*t)];
```

play



arccos method says 3392 Hz, not 4800 Hz for this example

Is $S > 2B$ here? No. S = 8192 Hz but B = 4800 Hz