Homework #2, ENGN 100-300, F15. Due **Thu. Oct. 8, in class**

Notes

- This is an individual assignment, not a group project. Refer to the course syllabus for the collaboration policies.
- For unstapled solutions, only the first page will be graded.
- In all problems below, assume we are using base-10 logarithm unless otherwise specified.

1. [0] The world population roughly doubled from 3 billion in 1960 to 6 billion in 1999. Suppose it continued at that rate, *i.e.*, doubling every 39 years.
   - Write an equation relating population p to year y.
   - What type of plot would yield a straight line for this type of population growth?
   - Determine the slope and intercept for that plot.

   Caution: year y is on the *horizontal* axis here!

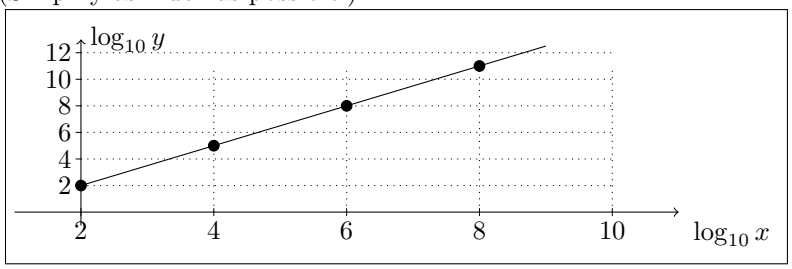
2. [5] A student drops a ball from the top of the Eiffel tower and measures the distance the ball has traveled (in meters) after 1, 2, 3, 5, and 7 seconds. She plots the data using a log-log plot. Determine an expression for and a numerical value for the intercept on that plot.

3. [15] If you put \$100 in a bank that pays a (compound) annual interest rate of 5% (you wish), then after one year (assuming no withdrawals, which is unlikely) your balance will be $100 \times 1.05 = \$105$, and after two years your balance will be $100 \times 1.05 \times 1.05 = \$110.25$, etc.
   - Write an equation that relates the balance b to the number of years n that the money is in the bank earning 5% interest.
   - If you want to plot balance (on the vertical axis) versus number of years and have the data points lie along a line, do you use a log-log or a semi-log plot?
   - Determine the slope for the appropriate plot.

4. [15] If a bank pays a (compound) annual interest rate of 2%, then each your balance increases by a factor of 1.02. Let's call this factor "f." You are comparing several banks with different interest factors (*e.g.*, 1.02, 1.05, etc.), and you are examining what your balance will be (assuming an initial deposit of \$100) after 20 years.
   - Write an equation that relates the balance b after 20 years to interest factor f.
   - If you want to plot balance after 20 years (on the vertical axis) versus the interest factor f and have the data points lie along a line, do you use a log-log or a semi-log plot?
   - Determine the slope for the appropriate plot.

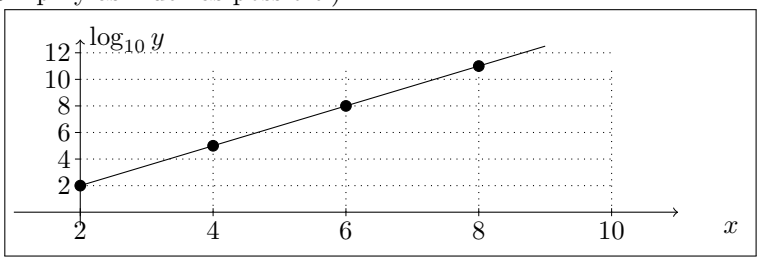
5. [5] An engineer numbers the 88 keys on a piano from 1 to 88 (from lowest to highest) and measures the (fundamental) frequency of each key. (The lowest note on a piano is an "A" with frequency 27.5 Hz, and this piano has the usual tuning.) She plots frequency versus key number using a semi-log plot. Determine the slope of the line in that plot.

6. [10] Based on the numerical data shown in the following plot, determine a formula relating y and x. (Simplify as much as possible.)



(over)

7. [10] Based on the numerical data shown in the following plot, determine a formula relating y and x. (Simplify as much as possible.)



Answer to #1:

- $p = 3 \times 2^{(y-1960)/39}$ (in billions)
  Note that for $y = 1960$: $p = 3 \times 2^0 = 3$ billion and for $y = 1999$: $p = 3 \times 2^1 = 6$ billion.
  Also note that the expression in the exponent is *unitless*: $\dfrac{y \text{ (in years)} - 1960 \text{ (year)}}{39 \text{ (years)}}$.
  (Any expression in an exponent *must* be unitless.)
- $\log_{10}(p) = \log_{10}(3) + \log_{10}\left(2^{(y-1960)/39}\right) = \log_{10}(3) + \frac{y-1960}{39}\log_{10}(2) = \left(\log_{10}(3) - \frac{1960}{39}\log_{10}(2)\right) + \frac{\log_{10}(2)}{39}y$
  So a semi-log plot is appropriate.
- The slope is $\frac{\log_{10}(2)}{39} \approx 0.00772$
  The intercept is $\log_{10}(3) - \frac{1960}{39}\log_{10}(2) \approx -14.65$