

Homework #9

This is for practice only – do not hand in.

1. Energy compaction in transform coding. Download the template `hw9_template.m` for use in this problem. This problem will again use the house image from homework #7. We will examine three different transforms: the DCT, the Hadamard and Haar transforms and we will use 8x8 blocks. The size 8 1D Haar transform matrix is:

$$Haar_8 = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ \sqrt{2} & \sqrt{2} & -\sqrt{2} & -\sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2} & \sqrt{2} & -\sqrt{2} & -\sqrt{2} \\ 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 2 \end{bmatrix}$$

The 1D Hadamard transform matrix can be determined from the Matlab `hadamard` command. The 2D DCT can be derived from Matlab's `dct` command. I would avoid using the `dct` function we created in homework #3 as that one was not an energy preserving transformation.

- Implement 8x8 transform for DCT, Hadamard, and Haar. Verify that each is energy preserving using a sample 8x8 image (sum squared energy in transform domain will be the same as sub squared energy in the image – or that $H^*H' = I$)
- Take the 8x8 transform for all 8x8 blocks in the image and store the transform coefficients in a 64x5400 array.
- Determine the variance of each of the coefficients and sort by magnitude.
- Plot the variances in increasing or decreasing order for each method. Use a log scale (`semilogy`) and put all three methods on one plot.
- Create a plot of the cumulative variance in the n highest coefficients for each method. Use a linear scale and put all three methods on one plot. Which method has the best energy compaction.
- Determine the number of bits that should be assigned to each coefficient for an average bit rate of 1 bit/pixel. Round to the nearest integer and disallow the assignment of negative bits/pixel. Which transform method results in the most coefficients coded with zero bits/pixel.