EECS 556, Imaging Processing, Douglas C. Noll 1/9/2003

Homework #1

Due Date: Jan. 16, 2002

1. [10] For the following systems, determine if the system is linear and if so determine the PSF. Also, determine if the system is space-invariant. Let a, b be non-zero, real numbers.
   a. $g(x,y) = \mathrm{S}[f(x,y)] = f(ax,ay)$
   b. $g(x,y) = \mathrm{S}[f(x,y)] = f(x\text{-}a,y\text{-}b)$
   c. $g(x,y) = \mathrm{S}[f(x,y)] = |f(x,y)|$

2. [5] State and prove the condition on $h(x,y)$ in order for a linear space invariant system to be rotationally invariant.

3. [10] Show or prove the following properties of 2D convolution.
   a. Shift property: $f(x,y)**d(x-x',y-y') = f(x-x',y-y')$
   b. Shift invariance: $f(x,y)**h(x,y) = g(x,y)$ implies that
      $f(x-x',y-y')**h(x,y) = g(x-x',y-y')$
   c. Circular symmetry: If $f(x,y)$ and $h(x,y)$ are circularly symmetric, then
      $f(x,y)**h(x,y)$ is also circularly symmetric.

4. [10] Find the <u>2D</u> Fourier transforms of:
   a. $\mathrm{sinc}(ax\text{-}b)$
   b. $\mathrm{sinc}(x\text{-}a)\mathrm{rect}(by)$
   c. $g_r(ar)$ given that $F\{g_r(r)\} = G(\boldsymbol{r})$.

5. [10] Determine the spatial resolution using, i) the Rayleigh criterion, ii) the Sparrow criterion, and iii) FWHM, for the following functions. Matlab's `fzero` may be useful here.
   a. $h(x) = \mathrm{sinc}(x)$
   b. $h(x,y) = \exp\left(-p\left(x^2 + y^2\right)\right)$

6. [100] Consider an imaging system with frequency response:
   $$H(\boldsymbol{r}) = \exp\left(-p(\boldsymbol{r}/16)^2\right) - \exp\left(-p(\boldsymbol{r}/4)^2\right).$$
   We would like to determine what the output image would be if the input image were
   $$f(x,y) = \mathrm{rect}(x/3,3y) + \mathrm{rect}(2x,y)$$
   One could solve this problem using convolution, but an easier way is be to use MATLAB's `fft2` command to compute the output image $g(x,y)$. Caution: you must be very careful with `fftshift` and your sample locations to get a correct answer. To work on this problem, you may wish to download the template file `h1template.m` from the web site.
   a. Display the real and the imaginary parts of your result as two distinct images using `subplot`. Display a `colorbar` to give the amplitude scale.
   b. Show $|F(u,v)|$ using `subplot` and `colorbar`.
   c. Is your resulting $g(x,y)$ exact or approximate at the sample locations? If not, why?