Homework #6

Due: 12/9/04

1. A region of the body has a 10cm thickness of muscle with 2cm of bone imbedded as shown in Figure 1. The densities (ρ) of muscle and bone are 1.0 and 1.75 g/cm^3, respectively.
   a. Calculate the x-ray transmission along paths through muscle alone and through muscle and bone for photon energies of 30 and 100 keV using Figure 2. Assume mono-energetic beams of photons.
   b. Determine the contrast between the background (muscle only) and muscle + bone. Which energy has the best contrast? ($C = \Delta N / \overline{N}_{background}$)
   c. Assuming that number of photons in I_0 is inversely proportional to the photon energy and that I_0 is fixed (e.g. the total energy transmitted is the same), which energy has the best contrast to noise ratio? ($C = \Delta N / \overline{N}_{background}$)

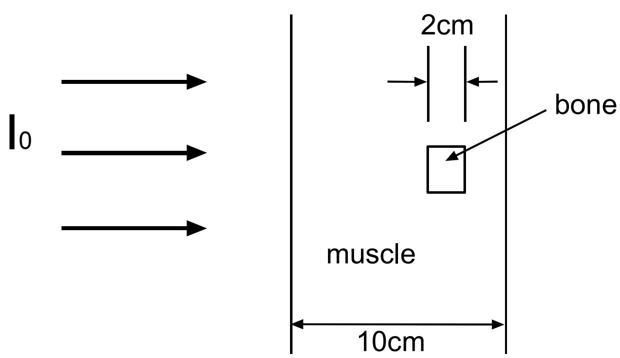


Figure 1. Object to be imaged

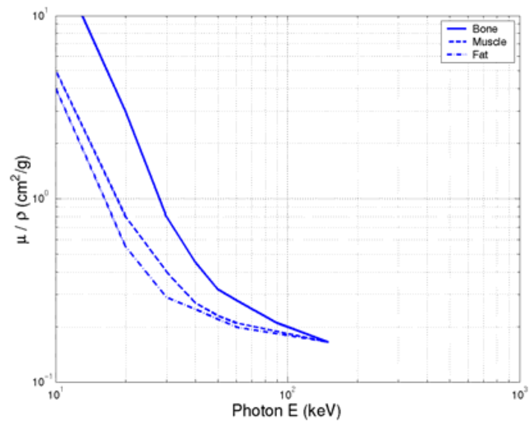
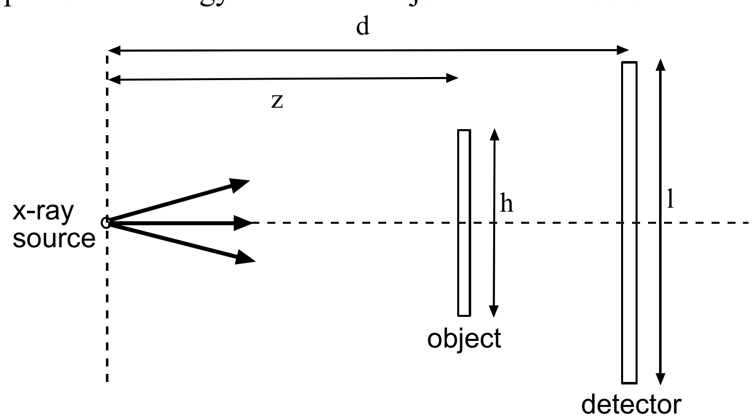


Figure 2. Mass attenuation coefficient of several tissues.

2. A source emits photons at energy E to a thin object as shown below.



   a. Fine the minimum energy of a Compton-scattered photon reaching the x-ray detector assuming single scattering events.
   b. What is this energy given E = 100keV, z = d/2, h = d/4, l = d/4.

3. Projection properties:
   a. If the areas of a function is $A = \iint f(x, y)dxdy$, find an expression for A in terms of $g_\theta(R)$.
   b. Suppose the project function is separable, e.g. $g_\theta(R) = h_R(R)h_\theta(\theta)$, show that this is not a valid project unless $h_\theta(\theta)$ is a constant.

4. Suppose we have a CT system with a parallel ray source. Let's assume that the detector system contains a scintillator that disperses light to the photodetectors in roughly a Gaussian pattern. That is, we can assume that the projection detected by the computer is the ideal project convolved with a Gaussian in the form $h(R) = \exp(-\pi R^2 / W^2)$. Find the reconstructed image $\hat{f}(x, y)$ in terms of the true image $f(x,y)$.