Homework #4
Due Date: Nov. 4, 2004

1. For magnetization vector $\mathbf{M}$ and applied field $\mathbf{B}_0$ with an angle $\alpha$ separating them, use the Bloch equations to show that the rate of precession of $\mathbf{M}$ around $\mathbf{B}_0$ is $\omega_0 = \gamma B_0$.

2. In class, we described quantum mechanical and classical properties of nuclear spin for the hydrogen nucleus, $^1\text{H}$. Let’s look at what changes if we are interested in imaging carbon-13, $^{13}\text{C}$. Assume that $\gamma_{^1\text{H}} = \frac{1}{4} \gamma_{^1\text{C}}$. Please describe the following for carbon in terms of the equivalent terms for hydrogen. Assume all other factors ($\mathbf{B}_0$, temperature, etc.) are the same.
   a. The difference in energy between spin-up and spin-down states, $\Delta E$. (Please describe $\Delta E$ for $^{13}\text{C}$ in terms of the $\Delta E$ for $^1\text{H}$.)
   b. The resonant frequency in the presence of applied field $B_0$, $\omega_0$.
   c. The number of excess nuclei that are spin-up vs. spin-down, $N_{\text{diff}}$.
   d. The length of the magnetic dipole for a single nuclear spin, $|\mu|$.
   e. The size of the equilibrium net magnetic dipole for 1 mole of the substance, $|\mathbf{m}|$.

3. Consider a magnetic dipole, $\mathbf{M}$, in the presence of an applied main magnetic field, $\mathbf{B} = B_0 \mathbf{k}$, where $\mathbf{k}$ is the unit vector in the z-direction. Assume that initially, $\mathbf{M}$ is at equilibrium, has length $M_0$ and is aligned with the main magnetic field (along the z-direction). Describe (or sketch) the position of $\mathbf{M}$ in a frame rotating at $\omega_0 = \gamma B_0$ for the following sequence of events. Please ignore the effects of any T1 or T2 relaxation.
   a. At equilibrium.
   b. A rotating magnetic field of strength $B_1$ and rotational frequency $\omega_0$ is applied for a period of time $\tau = \frac{2\pi}{4\gamma B_1}$.
   c. The main field is changed to $\mathbf{B} = (B_0 + \Delta B) \mathbf{k}$ for a period of time $T = \frac{\pi}{\gamma \Delta B}$.
   d. A rotating magnetic field of strength $B_1$ and rotational frequency $\omega_0$ is again applied for a period of time $\tau = \frac{2\pi}{4\gamma B_1}$. (Applied in the same direction as in b.)

4. Consider two materials, A and B with relaxation constants $(T_{1A}, T_{2A})$ and $(T_{1B}, T_{2B})$ respectively. Assume a 90 degree flip angle and at A, B have the same proton density.
   a. Determine the time that maximizes $\|M_{xy,A}(t) - M_{xy,B}(t)\|$.
   b. Determine the time that maximizes $|M_{z,A}(t) - M_{z,B}(t)|$.
   c. Assuming these materials was white matter and gray matter at 1.5 T, determine the repetition time (TR) that maximizes T1 contrast with a TE $= 0$ and determine the echo time (TE) that maximized T2 contrast for TR $= \infty$. 