Homework #2

Due Date: October 5, 2004

1. The purpose of this problem is to visualize the effect of frequency dependent dispersion in an ultrasound system. Use the following parameters: f_0 = 1.5 Mhz, c = 1500 m/s, attenuation is β = 0.1 $cm^{-1}MHz^{-1}$. Compute and plot the envelope of the received signals for ideal reflectors (R=1) at depths z=0, 4, 8, and 12 cm. Use the template Matlab program "templateh02_1.m" on the course web site. Do this for:
   a. $a(t) = rect(t f_0/3)$
   b. $a(t) = exp[-(t f_0/3)^2]$
   c. Comment on the differences in dispersion between these pulses. Which would make a better pulse to use in a practical ultrasound system? Why?

2. The power density in a pressure wave is given by $I \propto p^2$, where p is the amplitude of the pressure wave. Suppose we have a boundary between two media that have impedances Z_A and Z_B, where $Z_B = 5Z_A$.
   a. Calculate the reflected and transmitted power density through the boundary assuming the angle of incidence $\theta = 0$ and the power density of the incident wave is I_0. (Pressure wave goes from A to B.)
   b. Instead of going directly from medium A to medium B, we insert two intermediate media (C & D), where $Z_C = 2Z_A$ and $Z_D = 3Z_A$. (Pressure wave goes from A to C to D to B.) Calculate power density transmitted to medium B through these 3 boundaries assuming the angle of incidence $\theta = 0$. This is a form of impedance matching. Please neglect multiple reflections.

3. Consider a planar ultrasound transducer with no focusing. We are going to examine the transmit diffraction pattern in 1D (ignore y dimension). Assume that the transducer height is $2a$ = 10 mm ($C(x) = rect(x/2a)$), f_0 = 1.5 MHz, and c = 1500 m/s.
   a. What is the wavelength (λ) of the sound waves and what is the wavenumber (k)? At what approximate depth does the Fraunhoffer zone begin?
   b. Using the Fresnel approximation, determine the magnitude of the pressure response as a function of x_z for z = 25, 50, 100, 200 mm. Specifically, plot $|p(z,x_z)|$ vs. x_z. Use the template Matlab program "templateh02_3.m" on the course web site.
   c. For each depth, estimate the width of the response (e.g. the width at 50% of the peak response – know as FWHM for Full-Width Half-Maximum).
   d. Determine (analytically) the Fraunhoffer approximation for z = 100, 200 mm. Plot your analytical expression for $|p(z,x_z)|$ vs. x_z and compare your results to part 3(b).

4. Consider the ultrasound transducer of problem 3, but now we add focusing.
   a. Determine the complex aperture function for a focusing plane at z = 50 mm.
   b. Using the Fresnel approximation, determine the magnitude of the pressure response as a function of x_z for z = 25, 50, 100, 200 mm. Specifically, plot $|p(z,x_z)|$ vs. x_z. This will also use the template Matlab program "templateh02_3.m" on the course web site (case = 2).
   c. For each depth, estimate the FWHM of the response. Compare these to the results in problem 3(c).
   d. Determine (analytically – using the FT) and expression for $|p(z,x_z)|$ vs. x_z for z = 50 mm. Compare your results to part 4(b).