Homework #1

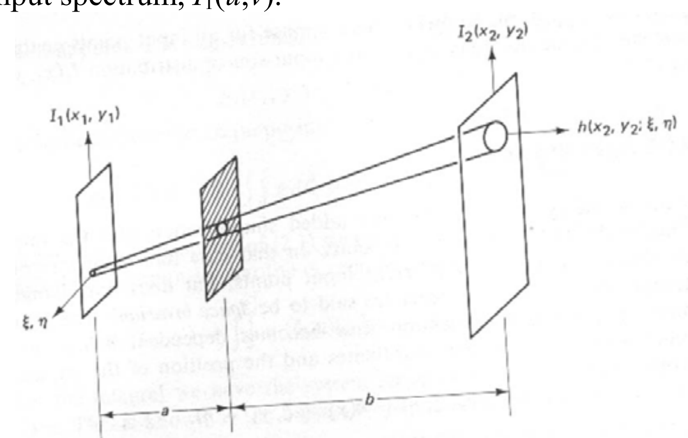
Due Date: Sept. 21, 2004

1. Consider the following systems, where $g(x,y) = S\{f(x,y)\}$:
   I. $S\{f(x,y)\} = f(ax,ay)$
   II. $S\{f(x,y)\} = f(x-a,y-b)$
   III. $S\{f(x,y)\} = \mathrm{sqrt}(\ f(x,y)\ )$
   IV. $S\{f(x,y)\} = \frac{1}{2}(f(x-a,y-b) + f(x+a,y+b))$

   where a, b are non-zero, real numbers. For each answer the following
   a. Is this system linear?
   b. Is this system space invariant?
   c. If the system is linear, determine the system impulse response.
   d. If the system is linear and space invariant, determine the Fourier transform of g in terms of the Fourier transform of f.

2. Let a, b be non-zero, real numbers. Find the 2D Fourier transforms of:
   a. rect(ax-b)
   b. rect(x-a)sinc(by)
   c. $\mathrm{circ}(r)\delta(x)$
   d. $\delta(r - r_0)$
   e. rect((r-a)/b), where a > b.
   f. $g_r(ar)$ [Let $F\{g_r(r)\} = G(\rho)$.]
   g. $\exp(-\pi(r/a)^2)$

3. A pinhole imaging system as shown below uses a circular pinhole of radius R. Using the geometry shown and assuming a constant collector efficiency (e.g. the pinhole is equally sensitive to all points on the image), find the output spatial frequency spectrum $I_2(u,v)$ in terms of the input spectrum, $I_1(u,v)$.



5. Derive (using either the definitions of the 2D delta function or properties of 1D delta functions) the following relationships:
   a. $f(x,y)\delta(x-a,y-b) = f(a,b)\delta(x-a,y-b)$
   b. $f(x,y)**\delta(x-a,y-b) = f(x-a,y-b)$
   c. $\delta(ax,by) = \delta(x,y)/|ab|$

6. Consider a charge coupled (CCD) imaging device that is used to sample image g(x,y). Assume that its FT $G(u,v)$ has no energy for spatial frequencies $\sqrt{u^2+v^2} \geq s_c$ and that the spacing of the collectors on the CCD is $\frac{1}{2s_c}$ in both the x and y directions. Also, assume that each collector is a square of size a by a, where $a < \frac{1}{2s_c}$. Finally, assume that all photons incident upon the collector are included in the sampled signal – that is, the sampled signal can be represented by the integration of the incident intensity over a square of size a by a.
   a. Show that sampled signal can be represented as:
   $$g_S(x,y) = [g(x,y)**c(x,y)]s(x,y)$$
   where $c(x,y)$ is the a by a collector function and $s(x,y)$ is an ideal sampling function.
   b. Specify a filter $H(u,v)$ that operates on the sampled spectrum $G_S(u,v)$, which will restore the original image