## Computed Tomography Project

Due Date: Dec. 14, 2004, 5 pm.
In this project you will implement and compare different methods of reconstructing an image from a set of projections. The methods to be implemented are: filtered-backprojection (FBP) and direct Fourier interpolation (FI). You will first need to generate a sinogram for the object described below - knowing the object will help debug your algorithm. Later you will apply one of the reconstruction methods to a subsampled sinograms and a sinogram with beamhardening.

We will assume a parallel ray imaging geometry presented below, the $\theta=0$ projection is shown here $\rightarrow$

The reconstructed image should be 128 by 128 pixels with a pixel size of 4 by 4 mm (field of view is 512 mm ). The number of radial samples is 128 , with $\Delta \mathrm{R}=4 \mathrm{~mm}$. The radial samples are symmetrically place as shown above (for 16 of the 128 lines). Note that the radial samples are at $R= \pm \Delta R / 2, \pm 3 \Delta R / 2$, $\pm 5 \Delta \mathrm{R} / 2, \ldots$ The angular sampling covers almost 180 degrees, i.e., $\theta_{\mathrm{k}}=\pi \mathrm{k} / \mathrm{N}_{\theta}$, where $\mathrm{k}=0,1,2, \ldots,\left(\mathrm{~N}_{\theta}-1\right)$ and $\mathrm{N}_{\theta}$ is the number
 of angular samples. Note the last angle is not at exactly 180 degrees. The object is shown here:


## Administrative items:

- You may work alone or with a partner for this project (any partner must be different from those you worked with in other projects). Please put your partner's name on your solutions.
- The partnering is not intended to be a "divide and conquer" approach - that is, you may not split up the problem and then share solutions. I expect you to do your own work, but when you get stuck, you may ask your partner or the instructor for help. Think of this as an exam, but one in which on may ask for help from one other person.
- Getting help from anyone other than the instructor or your partner or copying solutions (even from your partner) will be considered a violation of the engineering honor code. (see http://www.engin.umich.edu/org/ehc/hcode.html)
- Each person must hand in their own solution.
- Prepare a report describing the tasks below. Hand in answers to questions below, all requested plots/graphs/images and copies of your final ".m" files. For each question/task please write 2 or 3 sentences about what you did, what equation you implemented, and about the appearance of the resultant images, etc. Say something interesting!
- Title and label the axes on all plots and images.
- Files to get you started can be found in ftp://ftp.eecs.umich.edu/people/dnoll/bme516/ctproj/ or by using anonymous ftp to $\mathrm{ftp} . e e c s . u m i c h . e d u$ and then cd to people/dnoll/bme516/ctproj.
- If you are having problems, please come and see me (or send e-mail).
- Grading: $60 \%$ correct Matlab code, $30 \%$ report content, $10 \%$ plots, labeling axes, etc.


## Questions/Tasks:

1. Determine the required number of angular samples $\left(N_{\theta}\right)$. Let $N_{\theta}$ to be a multiple of 4 (part 8).
2. Synthesize a $\mathrm{N}_{\theta}$ by 128 sinogram for the object described above using analytical formulae for the path length through a circular object. The intensity projections using $\left(I_{0}=\right) n_{0}=100$ have been implemented for you in Matlab. Examine the image of the intensity projections and the sinogram. Describe the meaning of the brightness values (in the bar along side the images). Describe the features of the sinogram as they relate to the object.
3. Implement a simple backprojection algorithm and reconstruct the image without filtering. One easy way to do this in Matlab is to backproject at $\theta=0$ to a temporary image and then rotate that image to the appropriate angle of backprojection. Superimpose these temporary images to generate the final image.
4. Now filter each projection and plot the projection at $\theta=0$ before and after filtering. Please verify that the projection is real valued (there may be very small components in the imaginary part, say $\sim 10^{-6}$, which you should set to zero by using only the real part.
5. Generate the FBP image by back projecting the filtered sinogram. Set any negative pixels to zero and generate an image. Please describe any artifacts (imperfections) that you see in the images.
6. Use Matlab's griddata or interp2 functions to generate Fourier data for the object. Take the inverse Fourier transform to generate the image for the FI method. Please describe any artifacts that you see in the images.
7. Plot the profile through the image along the $y=0$ line for both methods (you may need to normalize to put on the same scale). Comment on any inaccuracies or artifacts.
8. Subsample the original sinogram by a factor of $4\left(\mathrm{~N}_{\theta} / 4\right.$ angular samples) and reconstruct the image. Reconstruct using the FBP approach and comment on any inaccuracies and artifacts.
9. We will now create noisy sinograms. To do this we will approximate Poisson distributed noise as Gaussian noise having a variance equal to the mean. Create a new projection (replacing the variable proj1) that approximates Poisson noise. Reconstruct using FBP and comment.
10. Repeat part 9 using $n_{0}=1,000$ and $n_{0}=10,000$. Comment on noise sensitivity in CT imaging.
11. We will now simulate the effects of beam hardening. To do this, will split the photon counts in half, where $1 / 2 n_{0}$ undergo the attenuations described in the Figure above (and same as part 2) and where the other $1 / 2 n_{0}$ undergo exactly $1 / 2$ of the attenuation. Create a new projection (replacing the variable proj1). Please do not add noise in this case. Reconstruct using FBP and plot the profile through the image along the $y=0$ line and comment on differences relative to part 7.
