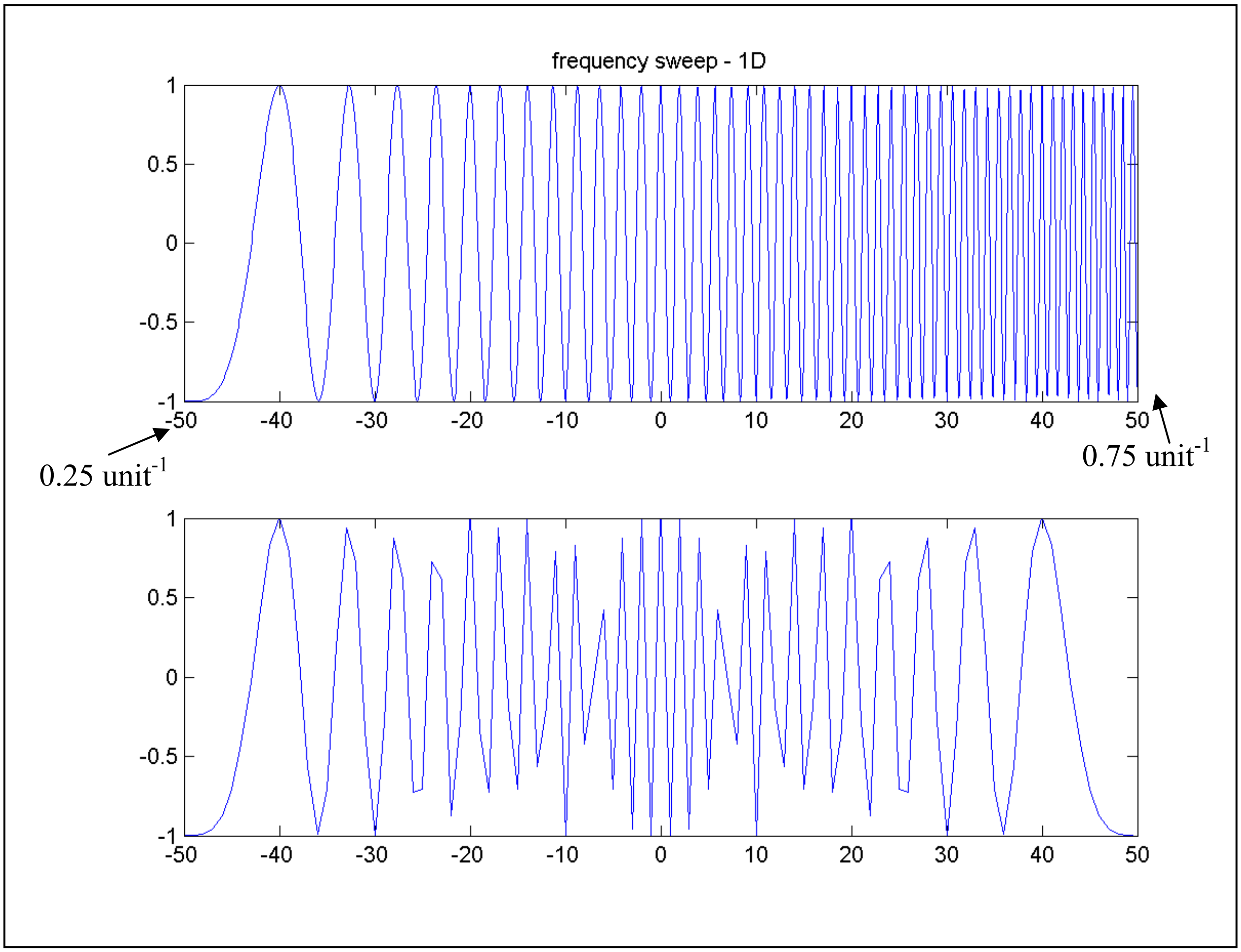
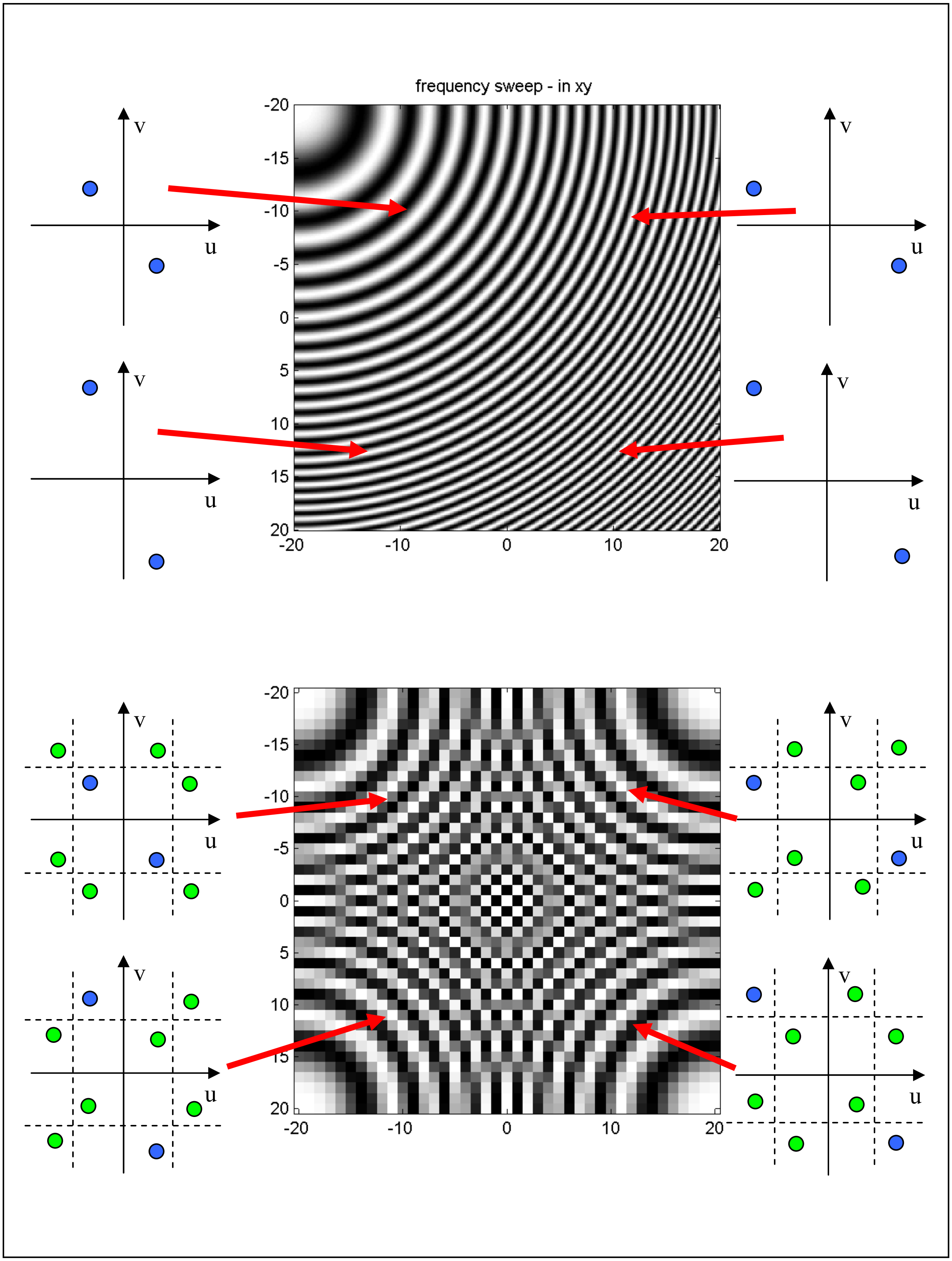
Aliasing in the Spatial Domain



In this example, the frequency is swept from a 0.25 to 0.75 cycles/unit distance as x goes from -50 to 50. At location zero, the frequency is 0.5 cycles/unit. In the upper plot, we see the original signal. In the lower plot, the signal is sampled with a space in of $\Delta x = 1$ unit ($f_s = 1$ unit^{-1})which means that all frequencies higher than $f_s/2 = 0.5$ unit^{-1} (anything to the right of 0) will be aliased. Indeed, as the frequency continues to go higher, the apparent frequency gets lower. The apparent frequency is $(f_s - f_i)$, where f_i is the local frequency.

In the second example (below), we extend this to two dimensions. Here we have a linear variation of frequencies in both x and y, that is, the x component of the frequencies varies from 0.25 to 0.75 cycles/unit and the y component of the frequencies varies from 0.25 to 0.75 cycles/unit. The upper image is the original signal and the lower image is the signal sampled at $\Delta x = \Delta y = 1$ unit. The dashed lines mark the +/- 0.5 unit^{-1} line the corresponds the Nyquist limit. Only the spectral components in the central box can be represented.

In the attached spectral plots, the blue dots correspond to delta functions of the true frequencies. The green dots are the spectral replicants in the case of sampling. Only the upper left quadrant has no aliasing. The upper left and lower right appear to have the same frequency, but the latter is aliased. The upper right and lower left also appear to have the same frequency, but from different aliasing mechanisms – aliasing of the x component and y-component, respectively.


frequency sweep - in xy
u
v
-20
-15
-10
-5
0
5
10
15
20