In this project you will implement and compare two methods of reconstructing an image from a set of projections. The methods to be implemented are: filtered-backprojection (FBP) and direct Fourier interpolation (FI). You will first need to generate a sinogram for the object described below – knowing the object will help debug your algorithm. Later you will apply one of the reconstruction methods to a subsampled object and a sinogram with beamhardening.

We will assume a parallel ray imaging geometry presented below, the 0=0 projection is shown:

![Parallel Ray Imaging Geometry Diagram](image)

The reconstructed image should be 128 by 128 pixels with a pixel size of 2 by 2 mm (field of view is 256 mm). The number of radial samples is 128, with $\Delta R = 2$ mm. The radial samples are symmetrically place as shown above (for 16 of the 128 lines). Note that the radial samples are at $R = \pm \Delta R/2 , \pm 3\Delta R/2 , \pm 5\Delta R/2 , \ldots$ The angular sampling covers almost 180 degrees, i.e., $\theta_k = \frac{\pi k}{N_\theta}$, where $k=0,1,2,\ldots,(N_\theta-1)$ and $N_\theta$ is the number of angular samples. Note the last angle is not at exactly 180 degrees. The object is show here:

![Object Description Diagram](image)

**Administrative items:**
- You may work alone or with a partner for this project (any partner must be different from those you worked with in other projects). Please put your partner’s name on your solutions.
- The partnering is not intended to be a “divide and conquer” approach – that is, you may not split up the problem and then share solutions. I expect you to do your own work, but when you get
stuck, you may ask your partner or the instructor for help. Think of this as an exam, but one in which on may ask for help from one other person.

- Getting help from anyone other than the instructor or your partner or copying solutions (even from your partner) will be considered a violation of the engineering honor code. (see http://www.engin.umich.edu/org/ehc/hcode.html)
- Each person must hand in their own solution.
- Prepare a report describing the tasks below. Hand in answers to questions below, all requested plots/graphs/images and copies of your final “.m” files. For each question/task please write 2 or 3 sentences about what you did, what equation you implemented, and about the appearance of the resultant images, etc. Say something interesting!
- Title and label the axes on all plots and images.
- Files to get you started can be found in ftp://ftp.eecs.umich.edu/people/dnoll/bme516/ctproj/ or by using anonymous ftp to ftp.eecs.umich.edu and then cd to people/dnoll/bme516/ctproj.
- If you are having problems, please come and see me (or send e-mail).

Questions/Tasks:

1. Determine the required number of angular samples ($N_\theta$). Let $N_\theta$ to be a multiple of 4 (part 8).
2. Synthesize a $N_\theta$ by 128 sinogram for the object described above using analytical formulae for the path length through a circular object. The parameters are described in the matlab template. Create an image of the sinogram. Describe features of the sinogram as they relate to the original object.
3. Implement a simple backprojection algorithm and reconstruct the image without filtering. One easy way to do this in Matlab is to backproject at $\theta = 0$ to a temporary image and then rotate that image to the appropriate angle of backprojection. Superimpose these temporary images to generate the final image.
4. Now filter each projection and plot the projection at $\theta = 0$ before and after filtering. Please verify that the projection is real valued (there may be very small components in the imaginary part, say $\sim 10^{-6}$, which you should set to zero by using only the real part.
5. Generate the FBP image by back projecting the filtered sinogram. Set any negative pixels to zero and generate an image. Please describe any artifacts that you see in the images.
6. Use matlab’s griddata or interp2 functions to generate Fourier data for the object. Take the inverse Fourier transform to generate the image for the FI method. Please describe any artifacts that you see in the images.
7. Plot the profile through the image along the $y=0$ line for both methods (you may need to normalize to put on the same scale). Comment on any inaccuracies or artifacts.
8. Subsample the original sinogram by a factor of 4 ($N_\theta/4$ angular samples) and reconstruct the image. Reconstruct using both approaches and comment on any inaccuracies and artifacts.
9. Modify the filter of part 4 to limit the frequency content of each projection. This filter should be selected to prevent aliasing using the FBP method (and can include appodization – e.g. a hanning or other filter, if you like). Compare the FBP images of the subsampled sinogram before and after filtering.
10. Now create a sinogram using only the first $1/4$ of the projections. Reconstruct using one of the approaches and comment on any inaccuracies and artifacts.
11. Load the mystery sinogram and examine it. Can you guess the object? Reconstruct the object by any method (the projection angles are uniformly spaced from 0 to 179 degrees) and generate the image. What is it? Describe/explain any artifacts you see.