Homework #2  
Due Date: October 10, 2006

1. The purpose of this problem is to visualize the effect of frequency dependent dispersion in an ultrasound system. Use the following parameters: \( f_0 = 1.5 \) MHz, \( c = 1500 \) m/s, attenuation is \( \beta = 0.1 \) cm\(^{-1}\)MHz\(^{-1}\). Compute and plot the envelope of the received signals for ideal reflectors (R=1) at depths \( z=0, 4, 8, \) and 12 cm. Use the template Matlab program “templateh02_1.m” on the course web site. Do this for:
   a. \( a(t) = \text{rect}(t f_0/3) \)
   b. \( a(t) = \exp[-(t f_0/3)^2] \)
   c. Comment on the differences in dispersion between these pulses. Which would make a better pulse to use in a practical ultrasound system? Why?

2. The power density in a pressure wave is given by \( I \propto p^2 \), where \( p \) is the amplitude of the pressure wave. Suppose we have a boundary between two media that have impedances \( Z_A \) and \( Z_B \), where \( Z_B = 5Z_A \).
   a. Calculate the reflected and transmitted power density through the boundary assuming the angle of incidence \( \theta = 0 \) and the power density of the incident wave is \( I_0 \). (Pressure wave goes from A to B.)
   b. Instead of going directly from medium A to medium B, we insert two intermediate media (C & D), where \( Z_C = 2Z_A \) and \( Z_D = 3Z_A \). (Pressure wave goes from A to C to D to B.) Calculate power density transmitted to medium B through these 3 boundaries assuming the angle of incidence \( \theta = 0 \). This is a form of impedance matching. Please neglect multiple reflections.

3. Consider a planar ultrasound transducer with no focusing. We are going to examine the transmit diffraction pattern in 1D (ignore \( y \) dimension). Assume that the transducer height is \( 2a = 10 \) mm ( \( C(x) = \text{rect}(x/2a) \) ), \( f_0 = 1.5 \) MHz, and \( c = 1500 \) m/s.
   a. What is the wavelength (\( \lambda \)) of the sound waves and what is the wavenumber (\( k \))? At what approximate depth does the Fraunhoffer zone begin?
   b. Using the Fresnel approximation, determine the magnitude of the pressure response as a function of \( x_z \) for \( z = 25, 50, 100, 200 \) mm. Specifically, plot \( |p(z,x_z)| \) vs. \( x_z \). Use the template Matlab program “templateh02_3.m” on the course web site.
   c. For each depth, estimate the width of the response (e.g. the width at 50% of the peak response – know as FWHM for Full-Width Half-Maximum).
   d. Determine (analytically) the Fraunhoffer approximation for \( z = 100, 200 \) mm. Plot your analytical expression for \( |p(z,x_z)| \) vs. \( x_z \) and compare your results to part 3(c).

4. Consider the ultrasound transducer of problem 3, but now we add focusing.
   a. Determine the complex aperture function for a focusing plane at \( z = 50 \) mm.
   b. Using the Fresnel approximation, determine the magnitude of the pressure response as a function of \( x_z \) for \( z = 25, 50, 100, 200 \) mm. Specifically, plot \( |p(z,x_z)| \) vs. \( x_z \). This will also use the template Matlab program “templateh02_3.m” on the course web site (case = 2).
   c. For each depth, estimate the FWHM of the response. Compare these to the results in problem 3(c).
   d. Determine (analytically – using the FT) and expression for \( |p(z,x_z)| \) vs. \( x_z \) for \( z = 50 \) mm. Compare your results to part 4(b).