Homework #4 Solutions

(Do not hand in, for practice only)

1. Consider a volume coil and a surface coil. Let the volume coil have sensitivity, $S_v(x) = 1$, and the surface coil have the following sensitivity pattern (as a function of distance from the coil):

   $S_s(x) = \dfrac{1}{\left(1+\left(\frac{x}{a}\right)^2\right)^{3/2}}$, where a is the coil radius.

   Let the noise variance of the volume coil be $\sigma_v^2 = 1$ and the noise variance of the surface coil be $\sigma_s^2 = 0.001 \cdot a^3$, where a is assumed to be in units of cm.

   a. For $a = 5$ cm, determine for which distance from the object surface it is advantageous (from a signal to noise ratio standpoint) to use the surface coil over the volume coil (and vice versa). SNR = (signal intensity)/σ, where σ is the noise standard deviation.

   b. For $a = 10$ cm, determine for which distance from the object surface it is advantageous to use the surface coil over the volume coil (and vice versa).

Solutions:

Assume that the signal strength (as a function of x) is equal to sensitivity $S(x)$ and the noise is equal to σ. The signal to noise ratio is then $S(x)/\sigma$. For the volume coil $S_v(x) = 1$ and $\sigma_v = 1$, therefore $SNR_v = 1$. For the surface coil, the SNR is

$$SNR_s = \frac{1}{(0.1\cdot a)^{3/2}\left(1+\left(\frac{x}{a}\right)^2\right)^{3/2}} = \frac{1}{\left((0.1\cdot a)\left(1+\left(\frac{x}{a}\right)^2\right)\right)^{3/2}}$$

To find the region where $SNR_s > SNR_v$, we merely need to find for which x that $SNR_s > 1$.

a. $a = 5$ cm, $SNR_s > SNR_v$, for $x < 5$ cm, that is if we are interested in a structure closer to the coil than 5 cm, it is preferred (from the SNR standpoint) to use the surface coil, otherwise the volume coil is better.

$$SNR_s = \frac{1}{\left(0.5\left(1+\left(\frac{x}{5}\right)^2\right)\right)^{3/2}} > 1$$

$$0.5\left(1+\left(\tfrac{x}{5}\right)^2\right) < 1$$

$$\left(\tfrac{x}{5}\right)^2 < 1$$

$$x < 5$$

b. $a = 10$ cm, $SNR_v > SNR_s$, for all non-zero values of x, therefore, the volume will always have better SNR. (No values of x satisfy the below relationship.)

$$SNR_s = \frac{1}{\left(1\left(1+\left(\frac{x}{10}\right)^2\right)\right)^{3/2}} > 1$$

$$\left(1+\left(\tfrac{x}{5}\right)^2\right) < 1$$

$$\left(\tfrac{x}{10}\right)^2 < 0$$

$$x^2 < 0$$

2. Consider 1 gram of gray matter brain tissue. Assume that the physiological parameters for this tissue at rest are:
   f = perfusion rate = 0.55 ml/min/g
   Oxygen extraction fraction (OEF) = 0.5
   Cerebral metabolic rate of oxygen (CMRO2) = a OEF f, where a is a constant
   V = Fractional blood volume = 0.05
   Q = Concentration of deoxyhemoglobin = b V OEF, where b is a constant
   $R2' = \frac{2Q}{3b}$ (in ms^{-1}), the relation component due to magnetic field perturbations
   $R2$ = 1/60 (in ms^{-1})

   a. What is the resting state $T2^*$?
   b. For TE = 30 ms, what is the image intensity (assume TR >> $T1$)?

   Now assume that the brain tissue becomes active resulting in an increase in the oxygen metabolism (CMRO2) of 5%. In order the satisfy the metabolic needs of the tissue, the perfusion rate (f) increases by 40%, which also results in a blood volume (V) increase of 20%.

   c. What is the new OEF? Has this gone up or down?
   d. What is the new Q? Has this gone up or down?
   e. What is the new $R2'$? Has this gone up or down?
   f. What is the new $T2^*$? Has this gone up or down?
   g. For TE = 30 ms, what is the image intensity (assume TR >> $T1$)? Has this gone up or down?

Solutions:

a. For this part, recall that decay rates add: $R2^* = R2 + R2'$. First, let's determine the resting state $R2' = \frac{2Q}{3b}$ = 2/3 V OEF = 2/3 * 0.05 * 0.5 = 1/60 ms^{-1}. $R2^* = R2 + R2'$ = 1/30 ms^{-1}. Or T2* = 30 ms.
b. Image intensity = r(1-exp(-TR/$T1$)exp(-TE/$T2^*$) = $r$exp(-1) = 0.3679r.

c. Using the equation for CMRO2, we solve for a = CMRO2*2/f. and substituting new values for flow and CMRO2, we get: 1.05*(CMRO2) = a OEF_{new} (f)*1.4 = CMRO2*2*OEF_{new}*1.4, and solving for OEF_{new} = 0.375. This has gone down.

d. $Q = b\ V\ OEF_{new}$ and V has increased 20% to 0.06 so Q = 0.0226 b. This has gone down from 0.025 b.

e. $R2' = \frac{2Q}{3b}$ = 2/3 V OEF = 2/3 * 0.06 * 0.375 = 1/66.667 ms^{-1}. This has gone down.

f. $R2^* = R2 + R2' \rightarrow$ T2* = 31.38 ms. This has gone up.

g. Image intensity = $r$exp(-TE/$T2^*$) = $r$exp(-1) = 0.3868r. This has gone up by 5.1%.