BME 483, Introduction to MRI 4/17/01
Douglas C. Noll Handout #10

Homework #3 Solutions
Due: 4/12/01

1. Consider a 1D object in the form $g(x) = m_0$ triangle(x/X), where X = 10 cm. Suppose we wish to image this object (in 1D) applying a 90 degree RF pulse followed by a gradient, G_x = 10 mT/m.
   a. What is the Fourier transform of $g(x)$?
   b. What is the space-frequency relationship (in Hz/cm) with the above G_x?
   c. Give an expression for the received signal, $s(t)$, after the RF pulse.
   d. What is the maximum spatial extent of the object? What is the maximum frequency component of the object (rotating frame)? What is the minimum required sampling rate, f_s, to prevent aliasing?
   e. Graphically, draw $s(t)$ and mark the locations of samples when they occur at the rate specified in part d.

Solutions

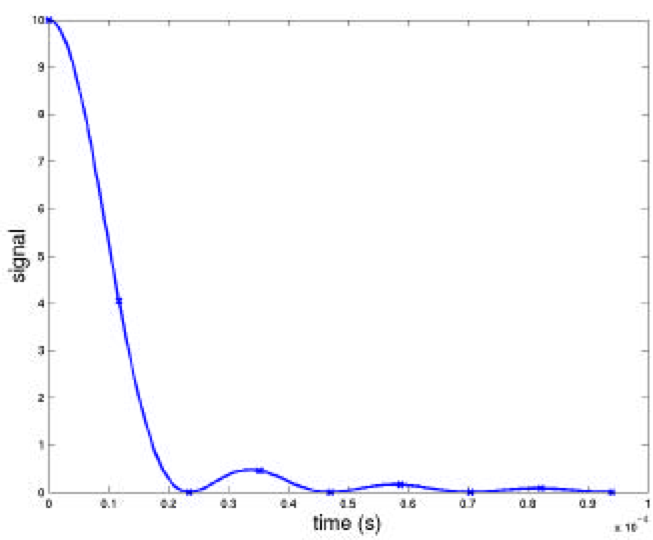
a. $F\{g(x)\} = |X| \text{ sinc}^2(Xu)$

b. In general, the frequency space relationship is $\Delta w(x) = gG_x x$ or in Hz it is $\Delta f(x) = (g/2p)G_x x$ = 42.58 MHz/T * 10^{-2} T/m * 10^{-2} m/cm = 4.258 kHz/cm.

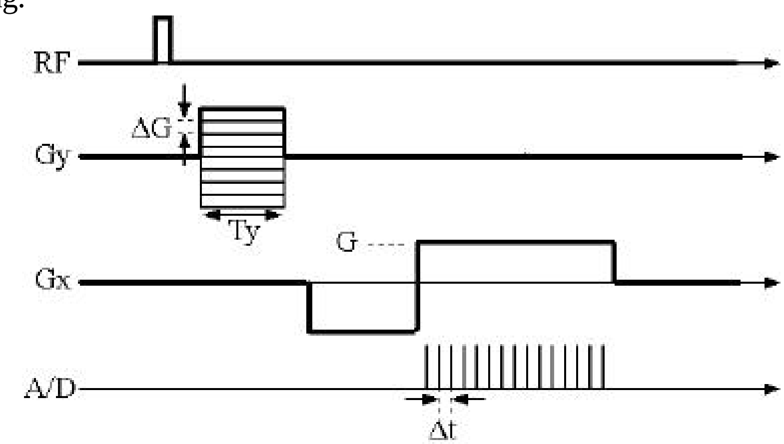
c. We will use the relationship $s(t) = F\{g(x)\}|_{u=k_x(t)}$. For this problem, $k_x(t) = \frac{g}{2p} G_x t$, therefore $s(t) = |X| \text{sinc}^2(X \frac{g}{2p} G_x t) = 10 \text{sinc}^2(42580t)$, for t measured in seconds.

d. The object $g(x)$ has maximum extent in x of X = 10 cm. The maximum frequency is f_{max} = 42.58 kHz. $f_s > 2{*}f_{\text{max}}$ = 85.16 kHz.

e. Plot shown here:



2. Consider an object with initial magnetization $m_0(x,y)$ = rect(x/X, y/Y).
   a. Determine the 2D Fourier transform of $m_0(x,y)$.
   b. For gradient waveforms $G_x(t) = A$ and $G_y(t) = 0$, give an expression for the received signal (this is similar to 1.c. above, but we have a 2D object).
   c. Determine the minimum sample spacing in both k_x and k_y to prevent aliasing of the object.
   d. In the spin-warp pulse sequence (below), determine T_y and G (in terms of ΔG, Δt and other parameters) so that k-space is sampled finely enough to prevent aliasing.



Solutions

a. $F\{m_0(x,y)\} = |XY|\,\mathrm{sinc}(Xu)\mathrm{sinc}(Yv)$

b. We will use the relationship $s(t) = F_{2D}\{m_0(x,y)\}|_{u=k_x(t),v=k_y(t)}$. For this problem, $k_x(t) = \frac{g}{2p}At; k_y(t) = 0$, therefore $s(t) = |XY|\mathrm{sinc}(X\,\frac{g}{2p}\,At)$.

c. The object $m_0(x,y)$ has maximum extent in x of $X/2$ and in y of $Y/2$. Using the relationships discussed in class, $\Delta k_x \le 1/2x_{\max}$ and $\Delta k_y \le 1/2y_{\max}$. Therefore $\Delta k_x \le \frac{1}{X}$ and $\Delta k_y \le \frac{1}{Y}$.

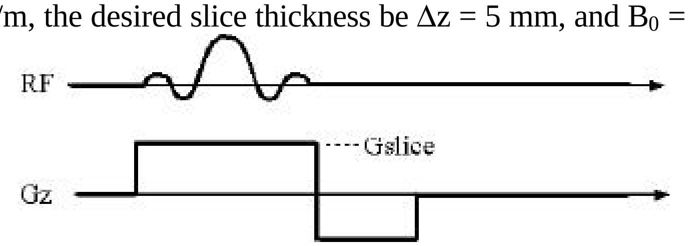
d. In a spin-warp pulse sequence, $\Delta k_x = \frac{g}{2p}G\Delta t$ and $\Delta k_y = \frac{g}{2p}T_y\Delta G$. Substituting into the results of part d., we get $G \le \frac{2p}{g\Delta tX}$ and $T_y \le \frac{2p}{g\Delta GY}$.

3. Consider the pulse sequence as shown above. Let T_y = 5 ms, and T_{read} = 20 ms. Suppose our desired field of views (FOVs) are $FOV_x = FOV_y$ = 20 cm and spatial resolution requirements are Δx = 1 mm and Δy = 2 mm. Determine the following:
   a. ΔG_y (in mT/m)
   b. $G_{y,max}$ (in mT/m)
   c. G_{read} (in mT/m)
   d. Δt (in ms)

Solutions:

a. $FOV_y = 1/\Delta k_y$ and $\Delta k_y = \gamma/2\pi\ T_y\, \Delta G_y$, thus $\Delta G_y = 2\pi/(\gamma\ T_y\ FOV_y)$ = 0.023 mT/m.
b. $\Delta y = 1/(2k_{y,max})$ and $k_{y,max} = \gamma/2\pi\ T_y\, G_{y,max}$, thus $G_{y,max} = 2\pi/(\gamma\ T_y\ 2\ \Delta y)$ = 1.17 mT/m.
c. $\Delta x = 1/W_{kx}$ and $W_{kx} = \gamma/2\pi\ T_{read}\, G_{read}$, thus $G_{read} = 2\pi/(\gamma\ T_{read}\ \Delta x)$ = 1.17 mT/m.
d. $FOV_x = 1/\Delta k_x$ and $\Delta k_x = \gamma/2\pi\ \Delta t\ G_{read}$ and $G_{read} = 2\pi/(\gamma\ T_{read}\ \Delta x)$, thus
   $\Delta t = T_{read}\ \Delta x\ /\ FOV_x$ = 0.1 ms (100 µs).

4. Consider the following slice selection pulse RF and gradient pulses. Let G_{slice} = 10 mT/m, the desired slice thickness be Δz = 5 mm, and B_0 = 1.5 T.



   a. Determine the center frequency and bandwidth of the RF excitation.
   b. Describe an RF pulse, $B_1(t)$, that has these features.

Solutions:

a. The center frequency for this RF pulse will be at $w_0 = gB_0$ or f_0 = 63.87 MHz. For a thickness Δz = 5 mm, the frequency bandwidth is $\Delta f = \Delta z^*\ G_{slice}\ \gamma/2\pi$ = 2.13 kHz.
b. The RF pulse, $B_1(t)$, has the form $B_1(t) = A\ \mathrm{sinc}(t/T)\exp(i2\pi f_0 t)$, which has a spectrum of $TA\ \mathrm{rect}(T(f-f_0))$. In the rotating frame, it is $B_1(t) = A\ \mathrm{sinc}(t/T)$, which has a spectrum of $TA\ \mathrm{rect}(Tf)$. This rect function has a width $1/T$, so for a bandwidth Δf, we set $T = 1/\Delta f$, where Δf is defined in part a. The amplitude must be selected for a flip angle of a. The area under the RF pulse is AT, so we set $gAT = a$ or $A = a/Tg = a\Delta f/g$. Therefore, $B_1(t) = a\Delta f/g\ \mathrm{sinc}(\Delta f\ t)\exp(i2\pi f_0 t)$.