1. Consider a volume coil and a surface coil. Let the volume coil have sensitivity, \( S_v(x) = 1 \), and the surface coil have the following sensitivity pattern (as a function of distance from the coil):
\[
S_s(x) = \frac{1}{\left(1 + (\frac{x}{a})^2\right)^{3/2}}, \text{ where } a \text{ is the coil radius.}
\]
Let the noise variance of the volume coil be \( \sigma_v^2 = 1 \) and the noise variance of the surface coil be \( \sigma_s^2 = 0.001 \ a^3 \), where \( a \) is assumed to be in units of cm.

a. For \( a = 5 \) cm, determine for which distance from the object surface it is advantageous (from a signal to noise ratio standpoint) to use the surface coil over the volume coil (and vice versa). SNR = (signal intensity)/\( \sigma \), where \( \sigma \) is the noise standard deviation.

b. For \( a = 10 \) cm, determine for which distance from the object surface it is advantageous to use the surface coil over the volume coil (and vice versa).

2. Consider 1 gram of gray matter brain tissue. Assume that the physiological parameters for this tissue at rest are:
\[
f = \text{perfusion rate} = 0.55 \text{ ml/min/g}
\]
\[
\text{Oxygen extraction fraction (OEF)} = 0.5
\]
\[
\text{Cerebral metabolic rate of oxygen (CMRO2)} = a \ \text{OEF} \ f, \text{ where } a \text{ is a constant}
\]
\[
V = \text{Fractional blood volume} = 0.05
\]
\[
Q = \text{Concentration of deoxyhemoglobin} = b \ V \ \text{OEF}, \text{ where } b \text{ is a constant}
\]
\[
R2^\prime = \frac{2Q}{3b} \text{ (in ms}^{-1}) \text{, the relation component due to magnetic field perturbations}
\]
\[
R2 = \frac{1}{60} \text{ (in ms}^{-1})
\]

a. What is the resting state \( T2^* \)?
b. For \( TE = 30 \) ms, what is the image intensity (assume \( TR >> T1 \))? 

Now assume that the brain tissue becomes active resulting in an increase in the oxygen metabolism (CMRO2) of 5%. In order to satisfy the metabolic needs of the tissue, the perfusion rate \( (f) \) increases by 40%, which also results in a blood volume \( (V) \) increase of 20%.

c. What is the new OEF? Has this gone up or down?
d. What is the new \( Q \)? Has this gone up or down?
e. What is the new \( R2^\prime \)? Has this gone up or down?
f. What is the new \( T2^* \)? Has this gone up or down?
g. For \( TE = 30 \) ms, what is the image intensity (assume \( TR >> T1 \))? Has this gone up or down?