4.22. (a) 
\[ \omega = \Omega T, \quad T = \frac{2\pi}{\Omega_0} \]

(b) To recover simply filter out the undesired parts of \( X(e^{j\omega}) \).

(c) 
\[ T \leq \frac{2\pi}{\Omega_0} \]

4.24

(a) 
(b) 
(c) 
(d)
4.25. (a) $x_s(t) = x_c(t) * s(t) \Rightarrow X_c(j\Omega) * s(j\Omega)$

(b) Since $H_d(e^{j\omega})$ is an ideal lowpass filter with $\omega_c = \frac{\pi}{4}$, we don't care about any signal aliasing that occurs in the region $\frac{\pi}{4} \leq \omega \leq \pi$. We require:

\[
\frac{2\pi}{T} - 2\pi \cdot 10000 \geq \frac{\pi}{4T} \\
\frac{1}{T} \geq \frac{8}{7} \cdot 10000 \\
T \leq \frac{7}{8} \times 10^{-4} \text{sec}
\]
4.41. (a) See figures below.
(b) From part (a), we see that

\[ Y_c(j\Omega) = X_c(j(\Omega - \frac{2\pi}{T})) + X_c(j(\Omega + \frac{2\pi}{T})) \]

Therefore,

\[ y_c(t) = 2x_c(t) \cos\left(\frac{2\pi}{T} t\right) \]
The system is linear, time-varying (due to downsampling), non-causal (due to \( \delta[n+1] \)), and stable.

(b) \[ T = \frac{\pi}{\Omega_N} = \frac{\pi}{2\pi \times 5000} = 10^{-4}\text{sec}, \quad \frac{L\omega_1}{T} = 2\pi \times 10^{4} \]
To avoid aliasing in $y_c(t)$:

\[
\frac{L \omega_1}{T} + \frac{2\pi}{T} \leq \frac{L\pi}{T} \\
\omega_1 = \frac{20\pi}{L} \\
20\pi + 2\pi = L\pi
\]

$L = 22$, $\omega_1 = 2\pi\left(\frac{10}{22}\right)$

(c) The Fourier transforms are sketched below.

\[
X_k(e^{j\omega})
\]
\[
\text{After } \uparrow L
\]
\[
\text{After LPF}
\]
\[
\text{After cosine modulation}
\]
\[
\text{After HPF}
\]
\[
Y(e^{j\omega})
\]
\[
Y_c(\Omega)
\]