1. Consider the systems shown below. Find $H_2(e^{j\omega})$, the frequency response of an LTI system, such that $y_2[n] = y_1[n]$ if the inputs to the systems are the same.

\[
x[n] \rightarrow H_1(e^{j\omega}) \rightarrow y_1[n]
\]

\[
x[n] \rightarrow H_2(e^{j\omega}) \rightarrow y_2[n]
\]

$Y_1(e^{j\omega})$ is $X(e^{2j\omega})H_1(e^{j\omega})$ downsampled by 2:

\[
Y_1(e^{j\omega}) = \frac{1}{2} \left\{ X(e^{j\omega/2})H_1(e^{j\omega/2}) + X(e^{j(\omega-2\pi)/2})H_1(e^{j(\omega-2\pi)/2}) \right\}
\]

\[
= \frac{1}{2} \left\{ X(e^{j\omega})H_1(e^{j\omega/2}) + X(e^{j(\omega-2\pi)})H_1(e^{j(\omega-2\pi)/2}) \right\}
\]

\[
= \frac{1}{2} \left\{ H_1(e^{j\omega/2}) + H_1(e^{j(\omega-\pi)}) \right\} X(e^{j\omega})
\]

\[
= H_2(e^{j\omega})X(e^{j\omega})
\]

\[
H_2(e^{j\omega}) = \frac{1}{2} \left\{ H_1(e^{j\omega/2}) + H_1(e^{j(\omega-\pi)}) \right\}
\]

2. Given the block diagram below

a) Develop the diagram into a complete signal flow diagram with full detail (direct I or II) for each subsystem.

b) Redraw it in cascade form with full detail.

c) Redraw it in a direct form with full detail.
3. Design a digital differentiator using the bilinear synthesis technique where the bilinear mapping function is \( s \rightarrow \frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}} \). (Remember that a perfect analog differentiator has a transfer function \( H_{\text{diff}}(s) = s \)) Sketch the pole/zero plot and the magnitude and phase characteristics. Explain why this is not a good design.

**ans:** There is a pole at -1 and a zero at 1. The Phase is flat at \( \pi/2 \) but the magnitude is only linear for low frequencies. For high frequencies, the magnitude increases too fast.

4. The filter described by a gain function and a zero plot below was created with the Remez exchange algorithm using Matlab. Its objective has a unity low frequency gain and high frequency gain of zero. Errors are weighted equally for all bands.
a) On this paper, mark the transition zone and the error bands.
b) How many points are there in the impulse function of this filter? 23
c) How many extrema are there in the filter function? Explain. 12

5. The following figure contains 8 parts. There are pairs of zero plots, phase plots, group delay plots and impulse response plots.
   a) Using a plot from each pair, list a self-consistent set by letter label.
   b) What is simple phrase that describes the set you have chosen.
   c) Is the other set consistent and if so, what is a phrase to describe it?
   d) How would the magnitude plots of the two sets compare?
a) CDFG
b) Linear phase filter
c) Minimum phase filter
d) Identical