Plane Wave Reflection and Refraction Involving a Finitely Conducting Medium

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1 Introduction and notations

In this work the structure of the electromagnetic field in a lossy conducting medium is studied in detail. The meaning of the complex angle of refraction is explained in terms of real parameters involved in this problem. The concepts are illustrated with a wave refracting from air into pure water at a frequency of 95 GHz and an incidence angle of 60°. The approach is similar to that used to derive refraction in many electromagnetic textbooks, such as Stratton [1], which has been cited in many of the textbooks published since. Our results agree with his for the real angle of the equiphase surface of the refracted fields, but we explore further subjects including the instantaneous and time-averaged Poynting vectors.

We consider medium 1 to be air (or some other lossless media) with a wave number denoted by

$$
\begin{aligned}
k_1 &= \omega\sqrt{\mu_1\epsilon_1} = \omega\sqrt{\mu_0\mu_{r1}\epsilon_0\epsilon_{r1}} \\
&= k_0\sqrt{\mu_{r1}\epsilon_{r1}}
\end{aligned} \tag{1}
$$

where $k_0 = \omega\sqrt{\mu_0\epsilon_0}$, and medium 2 to be a lossy dielectric with complex wave number denoted by

$$
\begin{aligned}
k_2 &= \omega\sqrt{\mu_2\epsilon_2} \\
&= \omega\sqrt{\mu_2\left(\epsilon_2' + i\frac{\sigma_2}{\omega}\right)} \\
&= \omega\sqrt{\mu_0\mu_{r2}\epsilon_0\left(\frac{\epsilon_2'}{\epsilon_0} + i\frac{\sigma_2}{\omega\epsilon_0}\right)} \\
&= k_0\sqrt{\mu_{r2}(\epsilon_{r2}' + i\epsilon_{r2}'')}
\end{aligned} \tag{2}
$$

where

$$
\epsilon_{r2} = \epsilon_{r2}' + i\epsilon_{r2}'' = \frac{1}{\epsilon_0}\left(\epsilon_2' + i\frac{\sigma_2}{\omega}\right) \tag{3}
$$

denotes the relative complex permittivity of the medium.

In the text that follows, a subscript of 1 will always denote a quantity associated with medium 1, and similarly, a subscript of 2 will always denote a quantity associated with medium 2. Also, a single prime will denote the real part of a quantity and a double prime will denote the imaginary part of a quantity. Thus, if z is a complex quantity, $z = z' + iz''$.

2 Transverse Electric

Let the incident wave have an electric field perpendicular to the plane of incidence. The field components of the incident, reflected and refracted waves can be written in the form

$$\mathbf{E}^i = A_{TE} e^{ik_1(\sin\theta_i x - \cos\theta_i z)} \hat{\mathbf{y}} \tag{4}$$

$$\mathbf{H}^i = \frac{k_1 A_{TE}}{\omega\mu_1} (\cos\theta_i \hat{\mathbf{x}} + \sin\theta_i \hat{\mathbf{z}}) e^{ik_1(\sin\theta_i x - \cos\theta_i z)} \tag{5}$$

$$\mathbf{E}^r = C_{TE} e^{ik_1(\sin\theta_r x + \cos\theta_r z)} \hat{\mathbf{y}} \tag{6}$$

$$\mathbf{H}^r = \frac{k_1 C_{TE}}{\omega\mu_1} (-\cos\theta_r \hat{\mathbf{x}} + \sin\theta_r \hat{\mathbf{z}}) e^{ik_1(\sin\theta_r x + \cos\theta_r z)} \tag{7}$$

$$\mathbf{E}^t = B_{TE} e^{ik_2(\sin\theta_t x - \cos\theta_t z)} \hat{\mathbf{y}} \tag{8}$$

$$\mathbf{H}^t = \frac{k_2 B_{TE}}{\omega\mu_2} (\cos\theta_t \hat{\mathbf{x}} + \sin\theta_t \hat{\mathbf{z}}) e^{ik_2(\sin\theta_t x - \cos\theta_t z)} \tag{9}$$

At the boundary at $z = 0$, the tangential components of $\mathbf{E}$ and $\mathbf{H}$ must be continuous. Hence

$$k_1 \sin\theta_i = k_1 \sin\theta_r = k_2 \sin\theta_t \tag{10}$$

or,

$$\left.\begin{aligned} \theta_i &= \theta_r \\ \frac{\sin\theta_t}{\sin\theta_i} &= \frac{k_1}{k_2} \end{aligned}\right\} \qquad \text{Snell's Law} \tag{11}$$

and

$$A_{TE} + C_{TE} = B_{TE} \tag{12}$$

$$\frac{k_1 \cos\theta_i}{\omega\mu_1}(A_{TE} - C_{TE}) = \frac{k_2 \cos\theta_i}{\omega\mu_2} B_{TE} \tag{13}$$

or,

$$\left.\begin{matrix} A_{TE} \\ C_{TE} \end{matrix}\right\} = \tfrac{1}{2}\left[1 \pm \frac{k_2 \cos\theta_t}{k_1 \cos\theta_i}\right] B_{TE} \qquad \text{Fresnel's Formula} \tag{14}$$

Fresnel's formula can be presented in several different forms. Expressing the ratio of k_2/k_1 in terms of $\sin\theta_t/\sin\theta_i$, the amplitudes A_{TE}, B_{TE}, and C_{TE} bear the following ratios:

$$A_{TE} : B_{TE} : C_{TE} = \sin(\theta_i + \theta_t) : (\sin(\theta_i + \theta_t) - \sin(\theta_i - \theta_t)) : \sin(\theta_i - \theta_t) \tag{15}$$

If we introduce the wave impedances

$$Z_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} \tag{16}$$

$$Z_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} \tag{17}$$

then, under the assumption that $\mu_1 = \mu_2$, we can state

$$\frac{k_1}{k_2} = \sqrt{\frac{\epsilon_1}{\epsilon_2}} = \frac{Z_2}{Z_1} \tag{18}$$

and Fresnel's formula can be written in the form

$$A_{TE} : B_{TE} : C_{TE} = (Z_2 \cos\theta_i + Z_1 \cos\theta_t) : 2Z_2 \cos\theta_i : (Z_2 \cos\theta_i - Z_1 \cos\theta_t) \tag{19}$$

$$= (Z_2/\cos\theta_t + Z_1/\cos\theta_i) : 2Z_2/\cos\theta_t : (Z_2/\cos\theta_t - Z_1/\cos\theta_i) \tag{20}$$

Sometimes it is convenient to use the transverse wave admittance defined by

$$\frac{H_x^i}{E_y^i} = \frac{-H_x^r}{E_y^r} = \frac{\cos\theta_i}{Z_1} = Y_1 \tag{21}$$

$$\frac{H_x^t}{E_y^t} = \frac{\cos\theta_t}{Z_2} = Y_2 \tag{22}$$

then (20) can be changed to

$$A_{TE} : B_{TE} : C_{TE} = (Y_1 + Y_2) : 2Y_1 : (Y_1 - Y_2) \tag{23}$$

So far the formulas are merely the formal solutions. In the first place, unless medium 2 is lossless ($\sigma_2 = 0$, or $\epsilon''_{r2} = 0$), the angle θ_t is a complex angle. The picture in Figure 1 is therefore only symbolic for a lossy medium when we are dealing with a complex θ_t. In particular, we are going to show that $\mathbf{H}^t$ is an elliptically polarized field, not transverse to $\mathbf{E}^t$. The plane of polarization corresponds to the plane of incidence, the x–z plane. For convenience, we introduce the notations

$$k_x = k_1 \sin\theta_i = k_2 \sin\theta_t \tag{24}$$

$$k_z = k_2 \cos\theta_t = k'_z + ik''_z = |k_z| e^{i\alpha} \tag{25}$$

$$\epsilon_r = \frac{\epsilon_2}{\epsilon_1} = \epsilon'_r + i\epsilon''_r = |\epsilon_r| e^{i\delta} \tag{26}$$

The parameters k_z' and k_z'' can be found by considering the relation

$$
\begin{aligned}
(k_2 \cos\theta_t)^2 = k_2^2\left(1-\sin^2\theta_t\right) &= k_2^2 - k_1^2 \sin^2\theta_i && (27)\\
&= k_1^2\left(\epsilon_r' + i\epsilon_r'' - \sin^2\theta_i\right) && (28)
\end{aligned}
$$

where again we assumed $\mu_1 = \mu_2$ and thus $k_2^2 = k_1^2(\epsilon_r' + i\epsilon_r'')$. Hence,

$$
(k_z' + ik_z'')^2 = k_1^2\left(\epsilon_r' - \sin^2\theta_i + i\epsilon_r''\right) \tag{29}
$$

Separating the real and imaginary parts of (29) and solving for k_z' and k_z'', we find

$$
\left.\begin{array}{l} k_z' \\ k_z'' \end{array}\right\} = k_1 \left[\frac{\sqrt{\left(\epsilon_r' - \sin^2\theta_i\right)^2 + \epsilon_r''^2} \pm \left(\epsilon_r' - \sin^2\theta_i\right)}{2}\right]^{1/2} \tag{30}
$$

Going back to the expression for $\mathbf{H}^t$ we have, in terms of the new notations,

$$
\begin{aligned}
H_x^t &= \frac{k_z B_{TE}}{\omega\mu_2} e^{i(k_x x - k_z z)}\\
&= \left[\frac{|k_z| B_{TE} e^{k_z'' z}}{\omega\mu_2}\right] e^{i(k_x x - k_z' z + \alpha)}\\
&= a_{TE} e^{i(\phi+\alpha)}, z \le 0 && (31)\\
H_z^t &= \frac{k_x B_{TE}}{\omega\mu_2} e^{i(k_x x - k_z z)}\\
&= \left[\frac{k_x B_{TE} e^{k_z'' z}}{\omega\mu_2}\right] e^{i(k_x x - k_z' z)}\\
&= b_{TE} e^{i\phi}, z \le 0 && (32)
\end{aligned}
$$

where $\phi = k_x x - k_z' z$,

$$
\begin{aligned}
a_{TE} &= \frac{|k_z| B_{TE} e^{k_z'' z}}{\omega\mu_2} && (33)\\
b_{TE} &= \frac{k_x B_{TE} e^{k_z'' z}}{\omega\mu_2} && (34)
\end{aligned}
$$

where we have arbitrarily chosen the phase of B_{TE} to be zero. The equiphase surface of both H_x^t and H_z^t correspond to

$$
\phi = k_x x - k_z' z = \text{ constant} \tag{35}
$$

The normal to the surface makes an angle η with the vertical axis (z-axis) where

$$\tan \eta = \frac{k_x}{k'_z} = \frac{k_x}{|k_z| \cos \alpha} \tag{36}$$

The constant amplitude surface corresponds to $z =$ constant, being parallel to the interface. The instantaneous value of H^t_x and H^t_z at a station (x, z) are given by

$$H^t_x(t) = a_{TE} \cos (\omega t - \phi - \alpha) \tag{37}$$

$$H^t_z(t) = b_{TE} \cos (\omega t - \phi) \tag{38}$$

They describe an elliptically polarized wave in x–z plane. The major axis of the ellipse makes a tilt angle γ with the vertical axis given by

$$\tan 2\gamma = \frac{2a_{TE}b_{TE}}{a^2_{TE} - b^2_{TE}} \cos \alpha \tag{39}$$

$$= \frac{2|k_z|k_x}{|k_z|^2 - k^2_x} \cos \alpha = \frac{2k'_z k_x}{|k_z|^2 - k^2_x} \tag{40}$$

and a minor axis to major axis ratio of $\tan \tau = \left(1 - \sqrt{1 - \sin^2 2\tau}\right) / \sin 2\tau$, where

$$\sin 2\tau = \frac{2a_{TE}b_{TE}}{a^2_{TE} + b^2_{TE}} \sin \alpha \tag{41}$$

$$= \frac{2k_x k''_z}{|k_z|^2 + k^2_x} \tag{42}$$

The magnetic field ellipse is shown in Figure 2.

The instantaneous Poynting vector associated with $\mathbf{E}^t$ and $\mathbf{H}^t$ are described by

$$S_x(t) = E^t_y(t) H^t_z(t) \tag{43}$$

$$= P_{TE} \cos^2 (\omega t - \phi) \tag{44}$$

$$= \tfrac{1}{2} P_{TE}(1 + \cos (2\omega t - 2\phi)) \tag{45}$$

$$S_z(t) = -E^t_y(t) H^t_x(t) \tag{46}$$

$$= Q_{TE} \cos (\omega t - \phi) \cos (\omega t - \phi - \alpha) \tag{47}$$

$$= \tfrac{1}{2} Q_{TE}(\cos \alpha + \cos (2\omega t - 2\phi - \alpha)) \tag{48}$$

where

$$P_{TE} = \frac{k_x B_{TE}^2 e^{2k_z'' z}}{\omega \mu_2} \tag{49}$$

$$Q_{TE} = \frac{-|k_z| B_{TE}^2 e^{2k_z'' z}}{\omega \mu_2} \tag{50}$$

The instantaneous Poynting vector $\mathbf{S}(t)$ is composed of a time-average component and a fluctuating component as follows: $\mathbf{S}(t) = \overline{\mathbf{S}} + \mathbf{s}(t)$.

The time-average values of $S_x(t)$ and $S_z(t)$ are therefore given by

$$\overline{S}_x = \tfrac{1}{2} P_{TE} \tag{51}$$

$$\overline{S}_z = \tfrac{1}{2} Q_{TE} \cos\alpha \tag{52}$$

The direction of $\overline{\mathbf{S}}$ makes an angle ζ_{TE} with the vertical axis with

$$\tan\zeta_{TE} = \frac{P_{TE}}{-Q_{TE}\cos\alpha} = \frac{k_x}{|k_z|\cos\alpha} = \frac{k_x}{k_z'} \tag{53}$$

which is the same as the angle η of the normal to the equiphase surface. The fluctuating part of $\mathbf{S}(t)$ is represented by

$$s_x(t) = \tfrac{1}{2} P_{TE} \cos(2\omega t - 2\phi) \tag{54}$$

$$s_z(t) = \tfrac{1}{2} Q_{TE} \cos(2\omega t - 2\phi - \alpha) \tag{55}$$

Their locus is an ellipse which has the same shape as the ellipse of the magnetic field vector except it has been rotated by 90° because

$$\frac{P_{TE}}{Q_{TE}} = \frac{-k_x}{|k_z|} = \frac{-b_{TE}}{a_{TE}} \tag{56}$$

The locus of $\mathbf{S}(t)$ is shown in Figure 3.

3 Transverse Magnetic

In the Transverse Magnetic case, where the magnetic field is perpendicular to the plane of incidence, the field components of the incident, reflected and transmitted (refracted) waves are

$$\mathbf{H}^i = A_{TM}\hat{\mathbf{y}}e^{ik_1(\sin\theta_i x - \cos\theta_i z)} \tag{57}$$

$$\mathbf{E}^i = \frac{-k_1 A_{TM}}{\omega\epsilon_1}(\cos\theta_i\hat{\mathbf{x}} + \sin\theta_i\hat{\mathbf{z}})e^{ik_1(\sin\theta_i x - \cos\theta_i z)} \tag{58}$$

$$\mathbf{H}^r = C_{TM}\hat{\mathbf{y}}e^{ik_1(\sin\theta_r x + \cos\theta_r z)} \tag{59}$$

$$\mathbf{E}^r = \frac{k_1 C_{TM}}{\omega\epsilon_1}(\cos\theta_r\hat{\mathbf{x}} - \sin\theta_r\hat{\mathbf{z}})e^{ik_1(\sin\theta_r x + \cos\theta_r z)} \tag{60}$$

$$\mathbf{H}^t = B_{TM}\hat{\mathbf{y}}e^{ik_2(\sin\theta_t x - \cos\theta_t z)} \tag{61}$$

$$\mathbf{E}^t = \frac{-k_2 B_{TM}}{\omega\epsilon_2}(\cos\theta_t\hat{\mathbf{x}} + \sin\theta_t\hat{\mathbf{z}})e^{ik_2(\sin\theta_t x - \cos\theta_t z)} \tag{62}$$

To satisfy the boundary condition at $z = 0$, we again must have

$$k_1\sin\theta_i = k_1\sin\theta_r = k_2\sin\theta_t \tag{63}$$

and

$$A_{TM} + C_{TM} = B_{TM} \tag{64}$$

$$\frac{k_1}{\epsilon_1}\cos\theta_i(A_{TM} - C_{TM}) = \frac{k_2}{\epsilon_2}\cos\theta_t B_{TM} \tag{65}$$

or

$$\left.\begin{array}{c} A_{TM} \\ C_{TM} \end{array}\right\} = \tfrac{1}{2}\left[1 \pm \frac{k_2\epsilon_1}{k_1\epsilon_2}\frac{\cos\theta_t}{\cos\theta_i}\right]B_{TM} \tag{66}$$

$$= \tfrac{1}{2}\left[1 \pm \frac{\sin\theta_t}{\sin\theta_i}\frac{\cos\theta_t}{\cos\theta_i}\right]B_{TM} \tag{67}$$

or

$$A_{TM} : B_{TM} : C_{TM} = (\sin 2\theta_i + \sin 2\theta_t) : 2\sin 2\theta_i : (\sin 2\theta_i - \sin 2\theta_t) \tag{68}$$

$$= \sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t) : 2\sin\theta_i\cos\theta_i : \sin(\theta_i - \theta_t)\cos(\theta_i + \theta_t) \tag{69}$$

Sometimes it is convenient to deal with the amplitudes of the electric field even for this case. Denoting these amplitudes by A, B and C, then

$$A = Z_1 A_{TM} \tag{70}$$

$$B = Z_2 B_{TM} \tag{71}$$

$$C = Z_1 C_{TM} \tag{72}$$

where

$$Z_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} \tag{73}$$

$$Z_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} \tag{74}$$

Then

$$A : B : C = A_{TM} : \frac{Z_2}{Z_1} B_{TM} : C_{TM} \tag{75}$$

Since, for $\mu_1 = \mu_2$,

$$\frac{Z_2}{Z_1} = \sqrt{\frac{\epsilon_1}{\epsilon_2}} = \frac{k_1}{k_2} = \frac{\sin\theta_t}{\sin\theta_i} \tag{76}$$

we have

$$A : B : C = \sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t) : 2\sin\theta_t\cos\theta_i : \sin(\theta_i - \theta_t)\cos(\theta_i + \theta_t) \tag{77}$$

$$= \tan(\theta_i + \theta_t) : \left[\frac{\tan(\theta_i + \theta_t)}{\cos(\theta_i - \theta_t)} - \frac{\tan(\theta_i - \theta_t)}{\cos(\theta_i + \theta_t)}\right] : \tan(\theta_i - \theta_t) \tag{78}$$

This form of Fresnel's formula is due to Sommerfeld [2].

We now consider the structure of $\mathbf{E}^t$, $\mathbf{H}^t$, and $\mathbf{S}$. The key formulas are

$$H_y^t = B_{TM} e^{i(k_x x - k_z z)} \tag{79}$$

$$= \left(B_{TM} e^{k_z'' z}\right) e^{i(k_x x - k_z' z)} \tag{80}$$

where the same notations are used, ie.,

$$k_x = k_1 \sin\theta_i = k_2 \sin\theta_t \tag{81}$$

$$k_z = k_2 \cos\theta_t = k_z' + i k_z'' = |k_z| e^{i\alpha} \tag{82}$$

Then,

$$E_x^t = \frac{1}{i\omega\epsilon_2}\frac{\partial H_y^t}{\partial z} = \left(\frac{-k_z B_{TM} e^{k_z'' z}}{\omega\epsilon_2}\right) e^{i(k_x x - k_z' z)} \tag{83}$$

$$= a_{TM} e^{i(k_x x - k_z' z + \alpha - \delta)} = a_{TM} e^{i(\phi + \alpha - \delta)} \tag{84}$$

$$E_z^t = \frac{-1}{i\omega\epsilon_2}\frac{\partial H_y^t}{\partial x} = \left(\frac{-k_x B_{TM} e^{k_z'' z}}{\omega\epsilon_2}\right) e^{i(k_x x - k_z' z)} \tag{85}$$

$$= b_{TM} e^{i(k_x x - k_z' z - \delta)} = b_{TM} e^{i(\phi - \delta)} \tag{86}$$

where

$$a_{TM} = \frac{-|k_z| B_{TM} e^{k_z'' z}}{\omega |\epsilon_2|} \tag{87}$$

$$b_{TM} = \frac{-k_x B_{TM} e^{k_z'' z}}{\omega |\epsilon_2|} \tag{88}$$

$$\epsilon_2 = |\epsilon_2| e^{i\delta} \tag{89}$$

The polarization ratio and the phase difference of E_x^t and E_z^t are the same as that of H_x^t and H_z^t given by (31) and (32). The locus of $\mathbf{E}^t$, therefore, has the same shape and the same inclination as the ellipse for $\mathbf{H}^t$ under the TE case, as shown in Figure 4. The nature of $\mathbf{S}(t)$, however, is different in this case because of the phase constant δ due to ϵ_2. The expressions for $S_x(t)$ and $S_z(t)$ are

$$\begin{aligned} S_x(t) &= -E_z(t) H_y(t) = P_{TM} \cos(\omega t - \phi) \cos(\omega t - \phi + \delta) \\ &= \tfrac{1}{2} P_{TM} \left[\cos\delta + \cos(2\omega t - 2\phi + \delta)\right] \end{aligned} \tag{90}$$

$$\begin{aligned} S_z(t) &= E_x(t) H_y(t) = Q_{TM} \cos(\omega t - \phi) \cos(\omega t - \phi - \alpha + \delta) \\ &= \tfrac{1}{2} Q_{TM} \left[\cos(\alpha - \delta) + \cos(2\omega t - 2\phi - \alpha + \delta)\right] \end{aligned} \tag{91}$$

where

$$P_{TM} = \frac{k_x B_{TM}^2 e^{2k_z'' z}}{\omega |\epsilon_2|} \tag{92}$$

$$Q_{TM} = \frac{-|k_z| B_{TM}^2 e^{2k_z'' z}}{\omega |\epsilon_2|} \tag{93}$$

The average values of $S_x(t)$ and $S_z(t)$ are therefore given by

$$\overline{S}_x = \tfrac{1}{2} P_{TM} \cos\delta \tag{94}$$

$$\overline{S}_z = \tfrac{1}{2} Q_{TM} \cos(\alpha - \delta) \tag{95}$$

The direction of $\overline{\mathbf{S}}$ makes an angle ζ_{TM} with the vertical axis with

$$\tan\zeta_{TM} = \frac{P_{TM}\cos\delta}{-Q_{TM}\cos(\alpha-\delta)} = \frac{k_x\cos\delta}{|k_z|\cos(\alpha-\delta)} = \frac{\tan\zeta_{TE}}{1+\tan\alpha\tan\delta} \tag{96}$$

which demonstrates that $\zeta_{TM} \neq \zeta_{TE}$ as given by (53). It is, therefore, not the same as the direction of the normal to the equiphase surface. In fact, ζ_{TM} is less than ζ_{TE}. The fluctuating part of $\mathbf{S}(t)$ is expressed by

$$s_x(t) = \tfrac{1}{2} P_{TM} \cos(2\omega t - 2\phi + \delta) \tag{97}$$

$$s_z(t) = \tfrac{1}{2} Q_{TM} \cos(2\omega t - 2\phi - \alpha + \delta) \tag{98}$$

Its locus is an ellipse which has the same shape as the one for the TE case because

$$\frac{P_{TM}}{Q_{TM}} = \frac{P_{TE}}{Q_{TE}} = \frac{-k_x}{|k_z|} \tag{99}$$

It also has the same inclination because the phase difference between $s_x(t)$ and $s_z(t)$ is still α. The locus of the TM Poynting vector is shown in Figure 5.

4 Application to the Air-Water Interface

For a given material, such as water, the complex dielectric constant is a function of frequency. Figure 6 shows the complex dielectric constant of pure water at 7°C from 1 GHz through 10 THz, as given by Ulaby, Moore and Fung [3]. The real part of the dielectric ranges from more than 84 to less than 5; the imaginary part ranges from essentially zero to more than 40; and δ is higher than 60° from 30 GHz to 80 GHz.

This common material produces a dramatic difference in the directions of power flow between TE and TM polarizations. For an incidence angle of 60°, the various angles associated with refraction given in the previous sections are shown in Figure 7. The difference in direction between TE and TM power flow is as great as 8°. The ratio of the minor axis to the major axis of the ellipse is higher than 0.125 from 91 GHz to 182 GHz.

It is interesting to note that

$$\tan(\zeta_{TE} + \zeta_{TM}) = \frac{2k_z' k_x - k_x k_z'' \tan\delta}{|k_z|^2 - k_x^2 - k_z''(k_z' \tan\delta + k_z'')} \tag{100}$$

which, when compared to (40), shows that $\gamma \approx \frac{1}{2}(\zeta_{TE} + \zeta_{TM})$ when $k_z'' \ll k_z'$, which is nearly always except for the case of total internal reflection, which corresponds to the condition $k_z'' > k_z'$, or, equivalently, $\epsilon_r' < \sin^2\theta_i$. For incidence from air onto water, this approximation is valid to better than a fraction of an arc second over all frequencies and incidence angles.

5 Conclusion

The complex angle of refraction does not represent the 'direction' of propagation of the refracted wave in a lossy medium. The direction of $\overline{\mathbf{S}}$ is a more meaningful measure of the direction of propagation. When $\mathbf{E}$ is perpendicular to the plane of incidence, the direction of $\overline{\mathbf{S}}$ is the same as the direction of the normal to equiphase front. When $\mathbf{H}$ is perpendicular to the plane of incidence, the direction of $\overline{\mathbf{S}}$ is different from the direction of the normal to equiphase front. In both cases the corresponding $\mathbf{H}$ and $\mathbf{E}$ are elliptically polarized in the plane of incidence.

References

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