Matched Subspace Detection Using Compressively Sampled Data
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Motivation and Objective

We model many signals in science and engineering using low-dimensional subspaces. We consider the problem of deciding whether a high-dimensional vector in $\mathbb{R}^d$ lies in a given $d$-dimensional ($d \ll n$) subspace $S$ by using only a few compressive measurements.

By leveraging random matrix theory, we show that a reliable test statistic can be constructed even with a few compressive measurements.

Problem Formulation

We seek to detect whether the unknown vector $v \in \mathbb{R}^n$ lies in an $d$ ($d \ll n$) dimensional subspace $S$ given only a small number of compressive measures of the form $x = Av + \xi$, where $A \in \mathbb{R}^{m \times n}$ ($m \leq n$) is the given sampling matrix and $\xi \in \mathbb{R}^m$ is additive noise.

Given $x = Av + \xi$ with $x, A$ and $U$ known, where $U \in \mathbb{R}^{n \times d}$ whose orthonormal columns span $S$.

Let $v = v_\perp + v_\parallel$ where $v_\parallel \in S$ and $v_\perp \in S^\perp$.

Can we obtain a reliable detector by looking at the projection residual in terms of undersampled data, i.e.,

$$T = \|(I - P_{\mathcal{U}})x\|^2_2 \overset{\text{H}_1}{\geq} \eta$$

where $P_{\mathcal{U}}$ denotes the projection operator onto the column space of $AU$.

Main Results

Assumptions

- The rows of sampling matrix $A$ are independent sub-gaussian random vectors with mean zero, $E\left[AA^T\right] = \frac{\sigma^2}{m}I_n$ and $\|A_i\|_2 \leq K$.
- The entries of noise vector are i.i.d sub-gaussian random variables with $E[\xi] = 0$, Cov$(\xi) = \sigma^2$ and $\|\xi\|_2 \leq K_1$.

Theorem 1 (Noiseless Data)

If $m > 2ed$, then with probability at least $1 - 3\exp[-\tau m]$ we obtain

$$\frac{m}{2en}\|v_\perp\|^2 \leq \|(I - P_{\mathcal{U}})Av\|^2 \leq \frac{m}{en}\|v_\perp\|^2$$

(The energy outside the subspace is preserved.)

Theorem 2 (Noisy Data)

Let $\eta = e(m - d)\sigma^2$, then the false positive rate is bounded:

$$P(T \geq \eta | \mathcal{H}_0) \leq \exp[-\tau(m - d)].$$

Additionally, suppose $m > 2ed$ and

$$\|v_\perp\|^2 \geq 4e(\epsilon + 2)(1 - d/m)\eta \sigma^2$$

holds for any $v \not\in S$, then the false negative rate is bounded:

$$P(T \leq \eta | \mathcal{H}_1) \leq \exp[-\tau(m) + \exp[-\tau(m - d)]$$

(A reliable detector can be obtained as long as $\|v_\perp\|^2$ scales with $n$.)

References

Matched subspace detection using compressively sampled data.
ICASSP, 2017.

Subspace detection of high-dimensional vectors using compressive sampling.