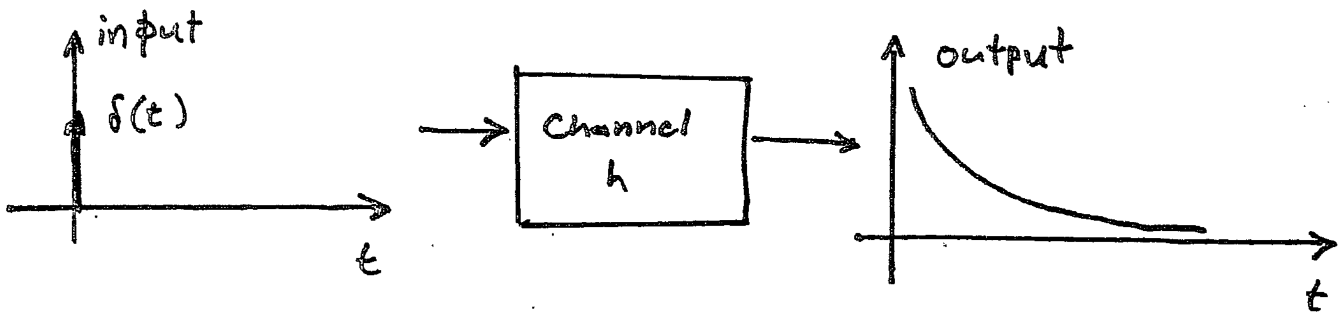
APPLICATION: CHANNEL ESTIMATION

Kalman Filtering for Time-Varying Channel Estimation

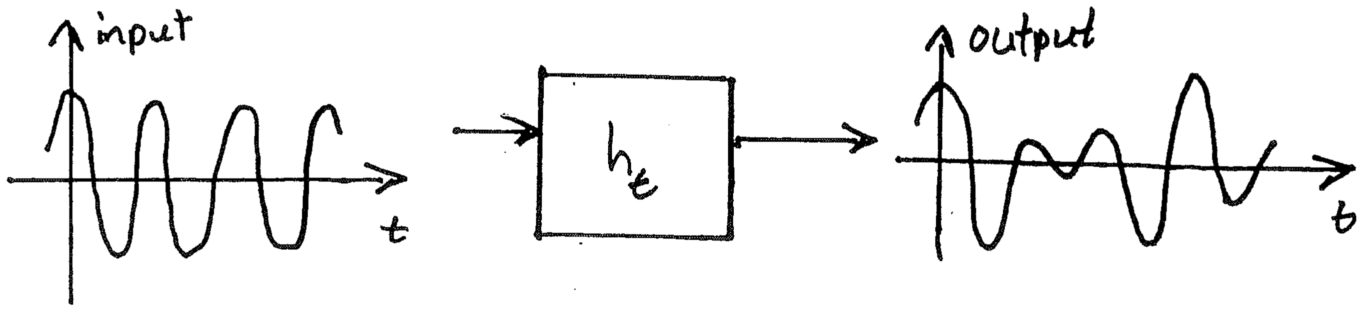
Multipath Communications Channel:



This effect is the result of many propagation paths, each of which delays and attenuates the input signal.

Additionally, the channel is time-varying due to movement of the source, receiver, and/or scattors. Therefore, the channel is acting like a linear time varying filter

Due to the time-varying nature of the channel, a sinusoidal input does not produce a pure sinusoid at the output:



Instead, the output is a narrowband process. Assuming the channel is relatively slowly varying (compare to the frequency of the input) we can view the input sinusoid as being amplitude modulated by the time-varying channel. This effect is refered to as <u>fading</u> and such channels are called <u>fading multipath</u> channels.

If we sample the output of the channel, then a very good model is the low-pass tapped delay line model:

$$y(n) = \sum_{k=0}^{p-1} h_n(k)\, v(n-k)$$

output: $y(n)$; input: $v(n-k)$; $h_n(k)$: impulse response depends on time n

This is simply an FIR filter with time-varying coefficients. In practice we wouldn't observe this perfect output, but rather a noise-corrupted version of it:

$$x(n) = \sum_{k=0}^{p-1} h_n(k)\, v(n-k) + w(n)$$

where $w(n)$ is observation noise.

The goal of channel estimation is to determine the linear time-varying filter $h_n(k)$ based on the input $v(n)$ and measured output $x(n)$. Is this possible?

Assume $v(n) = 0$ for $n < 0$. Then

$$x(0) = h_0(0)\,v(0) + h_0(1)\,v(-1) + w(0)$$
$$= h_0(0)\,v(0) + w(0)$$

$$x(1) = h_1(0)\,v(1) + h_1(1)\,v(0) + w(1)$$

$$x(2) = h_2(0)\,v(2) + h_2(1)\,v(1) + w(2)$$

$$\vdots$$

For each $n \geq 1$ we have two new parameters we must estimate!

Even in the absense of measurement noise we have more unknowns than equations and we can't determine the filter.

What can we do?

Well, suppose that the filter weights are not changing too rapidly from sample to sample. This is known as a <u>slow-fading</u> channel model.

Probabilistically, we can view the slowly varying channel as vector-valued Gauss-Markov process:

$$\underline{h}_{n+1} = A\,\underline{h}_n + \underline{u}_n$$

where $\underline{h}_n = [h_n(0), \ldots, h_n(p-1)]^T$, A is a pxp matrix designed to reflect the correlation expected between filter weights at different time samples, and $\underline{u}_n$ is a white Gaussian noise vector process with covariance Q.

That is,

$\ldots, \underline{u}_{n-1}, \underline{u}_n, \underline{u}_{n+1}, \ldots$ are iid vectors

and

$$\underline{u}_n \sim N(\underline{0}, Q)$$

A standard simplifying assumption is to assume that A and Q are diagonal $\Rightarrow$ filter weights are uncorrelated with each other.

This is called an <u>uncorrelated scattering</u> model.

The measurement/observation model in vector form is

$$x_n = \underline{v}_n^T \underline{h}_n + w_n$$

where $\underline{v}_n = [v(n), v(n-1), \ldots, v(n-p+1)]^T$.

With this notation and our Gauss-Markov model for the time-varying filter, we can now devise a Kalman filter to estimate and <u>track</u> the channel.

In the case we have the

<u>state equation</u>:

$$\underline{h}_{n+1} = A\,\underline{h}_n + \underline{u}_n \,, \quad n \geq 0$$

(with $\underline{h}_n$ in place of $\underline{s}_n$ now)

Furthermore, assume that

$$\underline{h}_0 \sim N(\underline{0}, R_0)$$

with R_0 also diagonal.

<u>Measurement equation</u>:

$$x_n = \underline{v}_n^T \underline{h}_n + w_n$$

Note that $\underline{v}_n$ is <u>known</u>, but <u>time-varying</u>. In our earlier discussion the H matrix of the observation model was <u>constant</u>. The Kalman filter still is applicable here, we just replace H with $\underline{v}_n^T$.

Kalman Filter

$$(\underline{s}_n = \underline{h}_n, \quad H = \underline{v}_n^T, \quad B = I)$$

$$\underline{\hat{h}}_{n|n-1} = A\, \underline{\hat{h}}_{n-1|n-1}$$

$$M_{n|n-1} = A\, M_{n-1|n-1} A^T + Q$$

$$K_n = \frac{M_{n|n-1} \cdot \underline{v}_n}{\underline{v}_n^T M_{n|n-1} \underline{v}_n + \sigma_w^2}$$

$$\underline{\hat{h}}_{n|n} = \underline{\hat{h}}_{n|n-1} + K_n (x_n - \underline{v}_n^T \underline{\hat{h}}_{n|n-1})$$

$$M_{n|n} = (I - K_n \underline{v}_n^T) M_{n|n-1}$$

Example 1 (p = 2)

Slow-Fading Model Parameters

$$A = \begin{bmatrix} 0.999 & 0 \\ 0 & 0.999 \end{bmatrix}$$ ← state transition matrix

$$Q = \begin{bmatrix} 0.0001 & 0 \\ 0 & 0.0001 \end{bmatrix}$$ ← driving noise covariance

to reflect little or no knowledge about the initial state of the channel

$$\underline{h}_0 \sim N\left(\underline{0}, \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}\right).$$

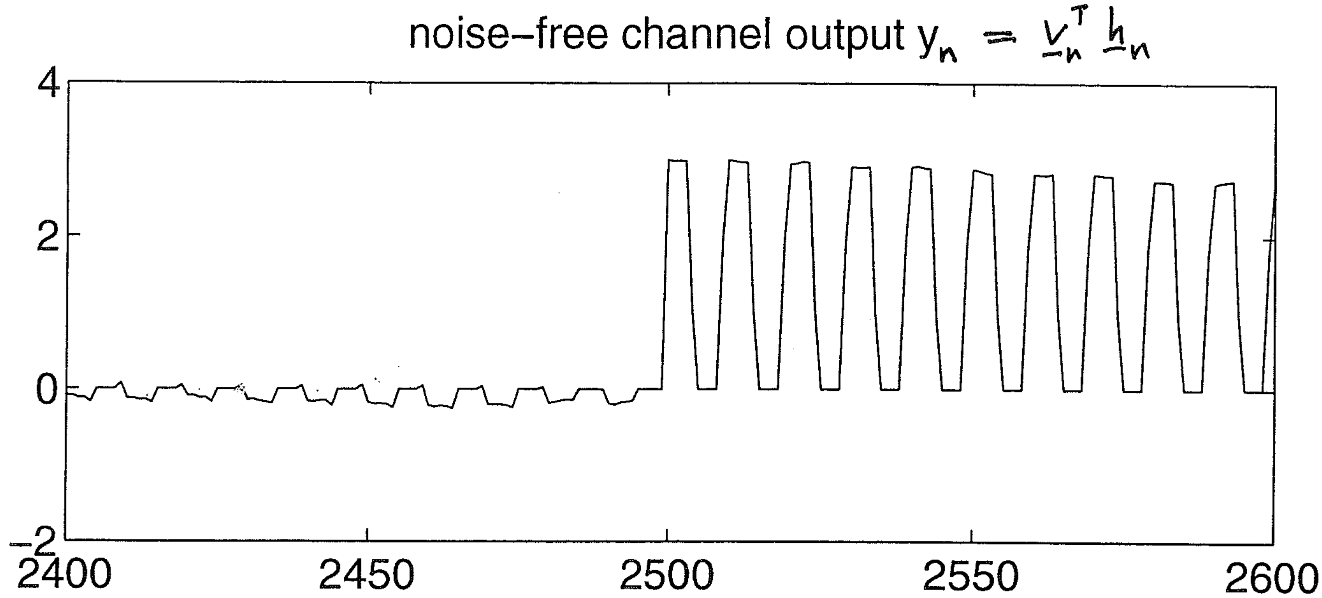
Channel model:

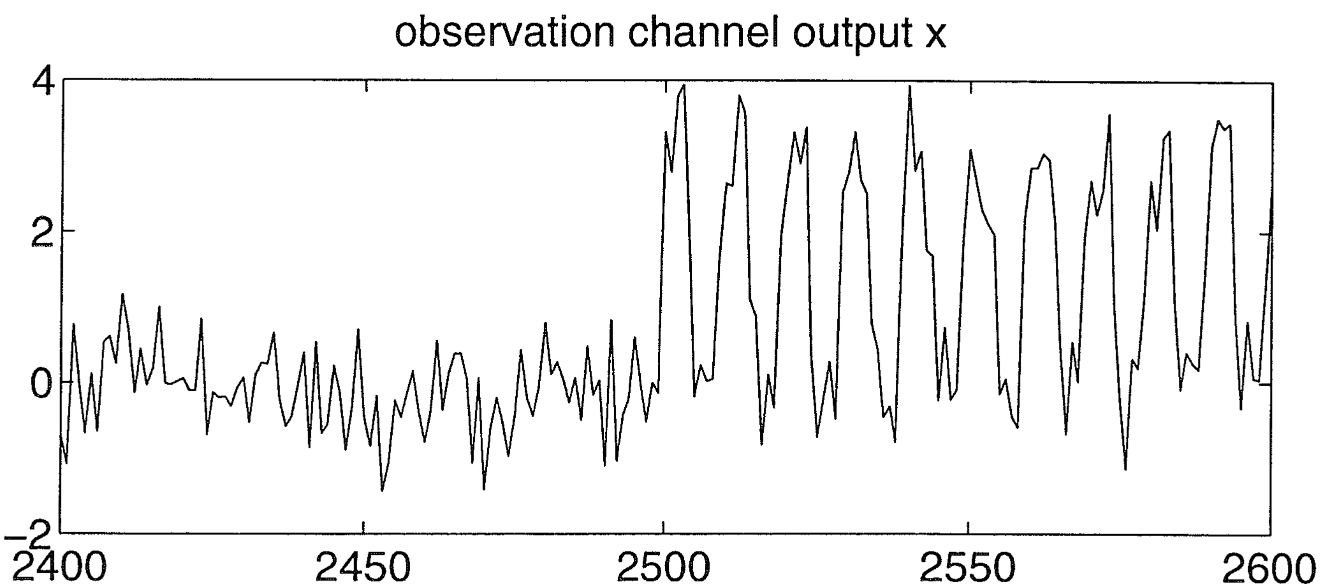
$$x(n) = h_n(0)\,v(n) + h_n(1)\,v(n-1) + w(n)$$

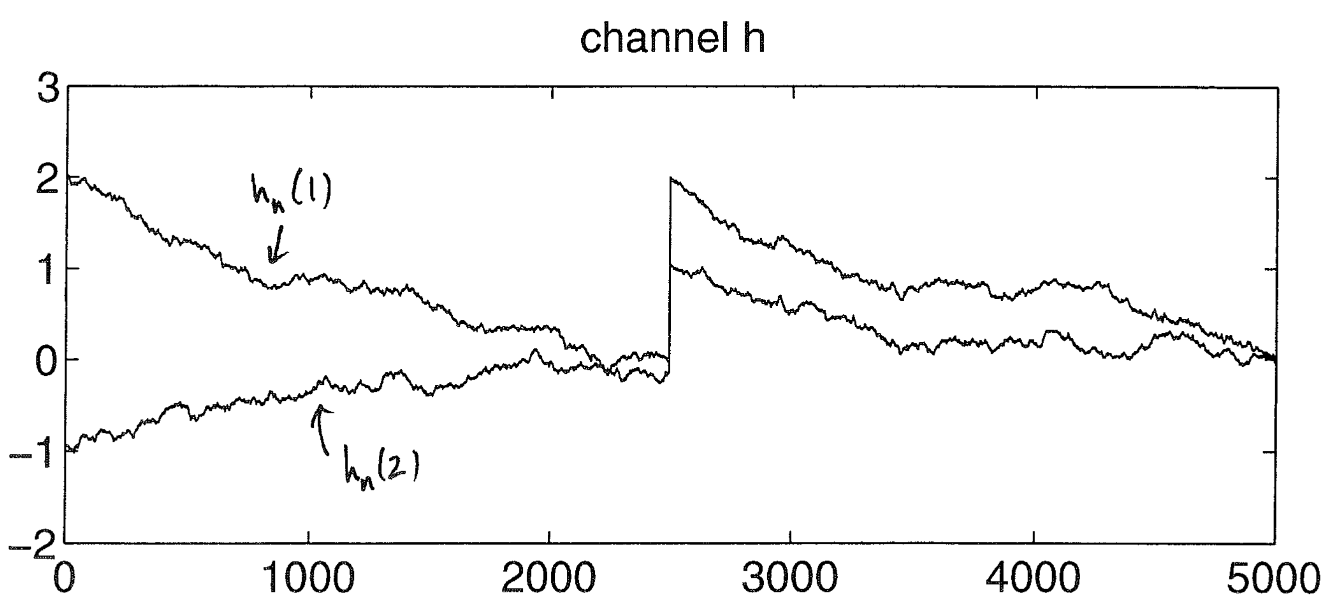
$v(n)$ = input to channel, known square wave

$h_n(k)$ = time-varying channel model (linear time-varying FIR filter)

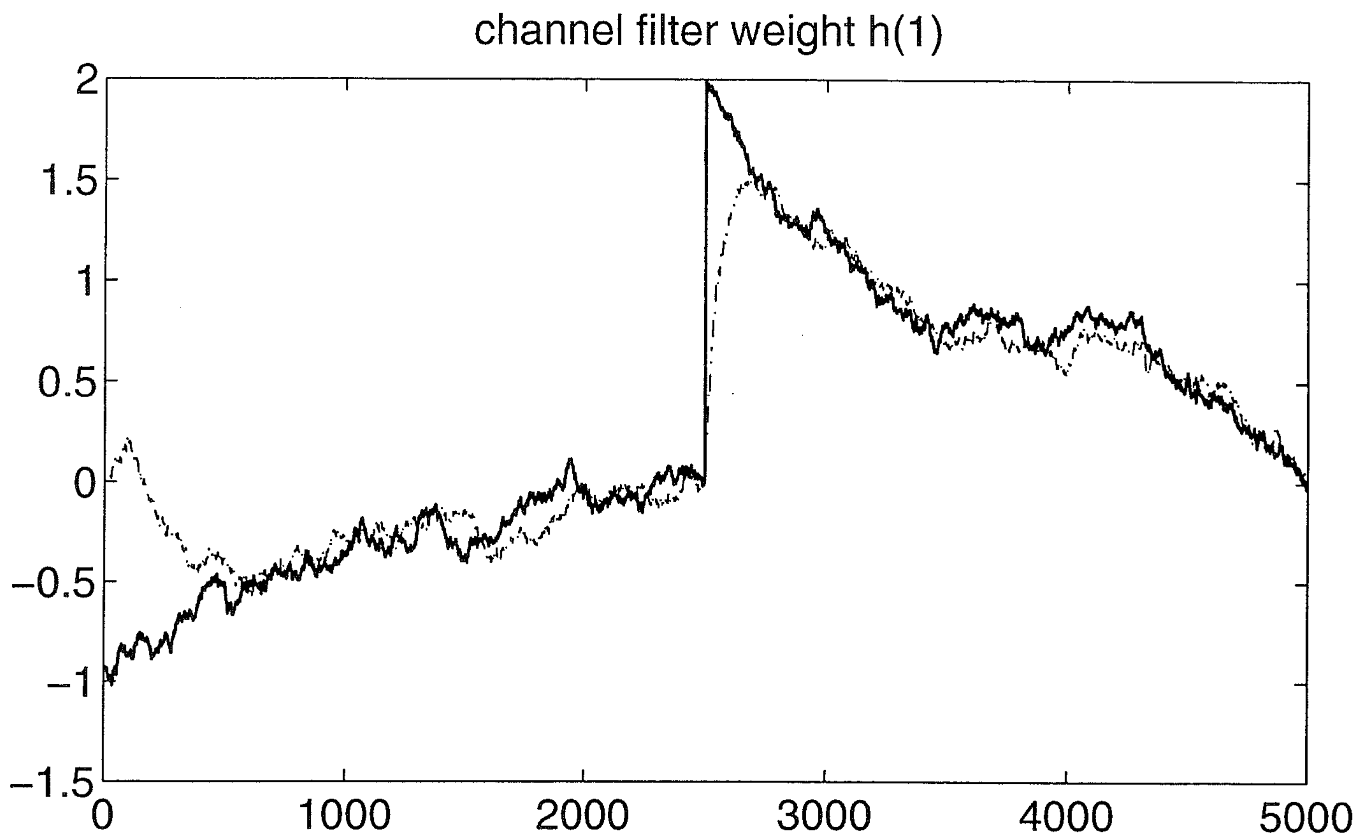
$w(n)$ = white Gaussian observation noise

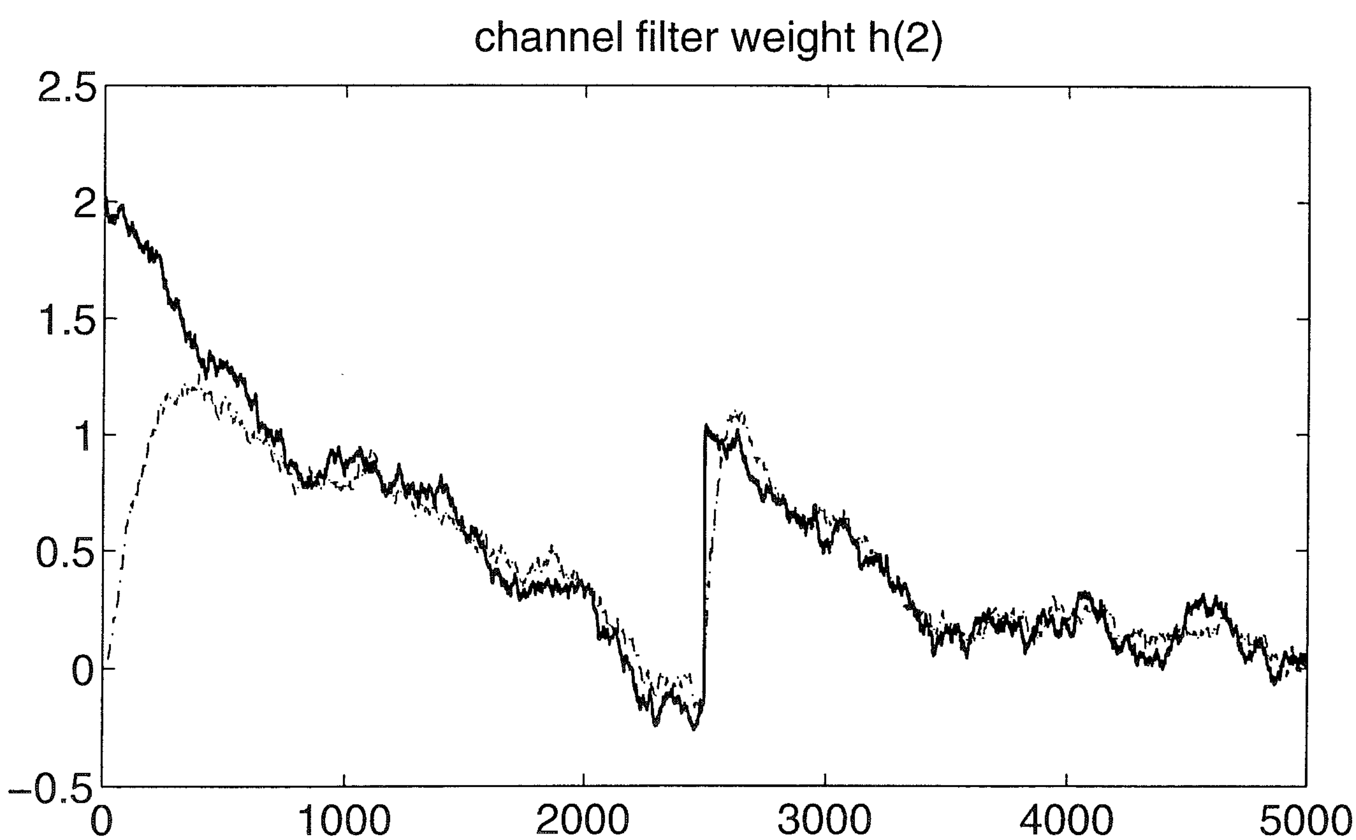






↑ filter changes drastically
(e.g., car drives out of a tunnel)


channel filter weight h(1)
2
1.5
1
0.5
0
−0.5
−1
−1.5
0
1000
2000
3000
4000
5000


channel filter weight h(2)
2.5
2
1.5
1
0.5
0
−0.5
0
1000
2000
3000
4000
5000