$$\boxed{\begin{array}{c}\text{Bearing}\\ \text{Estimation}\end{array}}$$

Array Processing

Suppose we want to determine the direction of propagation, or <u>bearing</u>, of a signal.

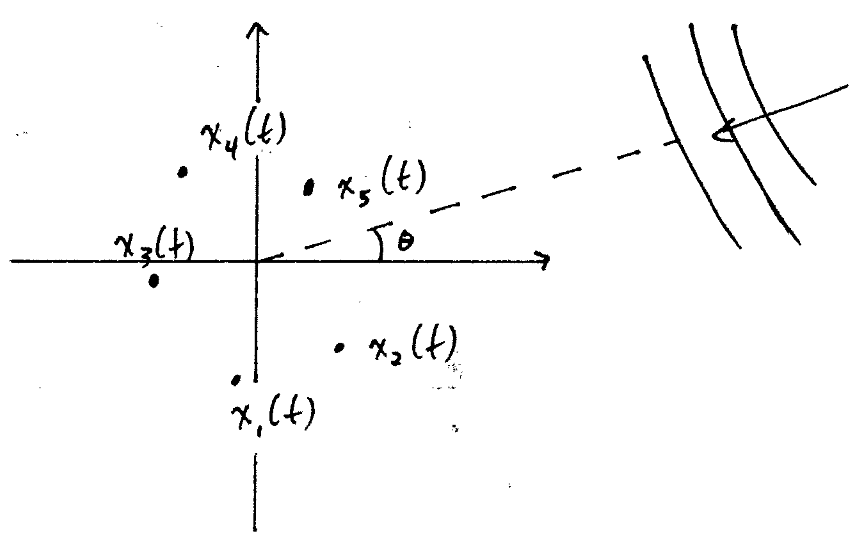
Directional sensor

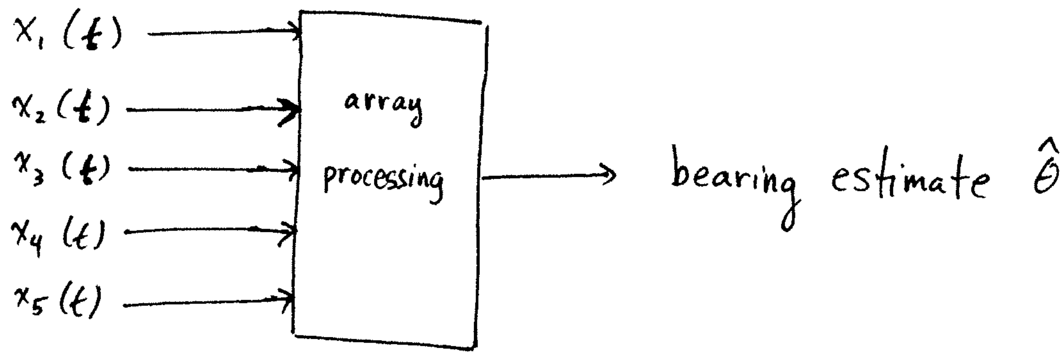
- expensive
- one direction at a time

parabolic dish

Array of omnidirectional sensors

- cheap
- multiple directions/sources at a time
- requires signal processing

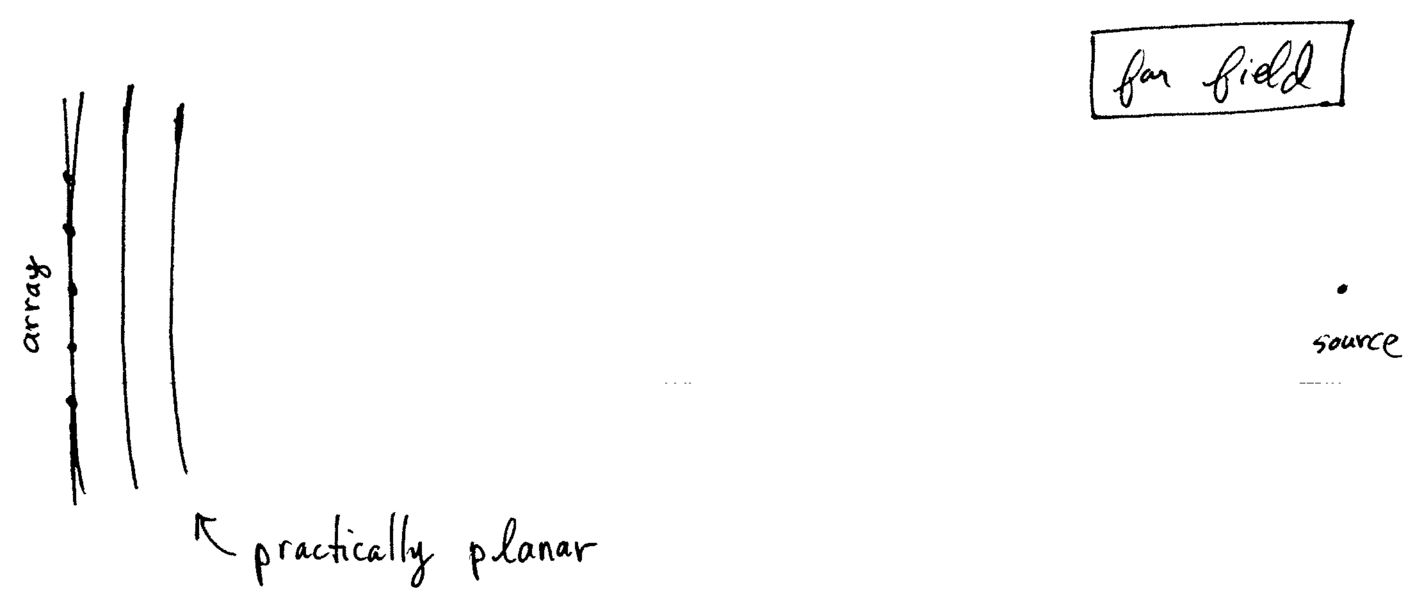




Assumptions on the signal source:

<u>near-field</u> $\Rightarrow$ spherical waves at array

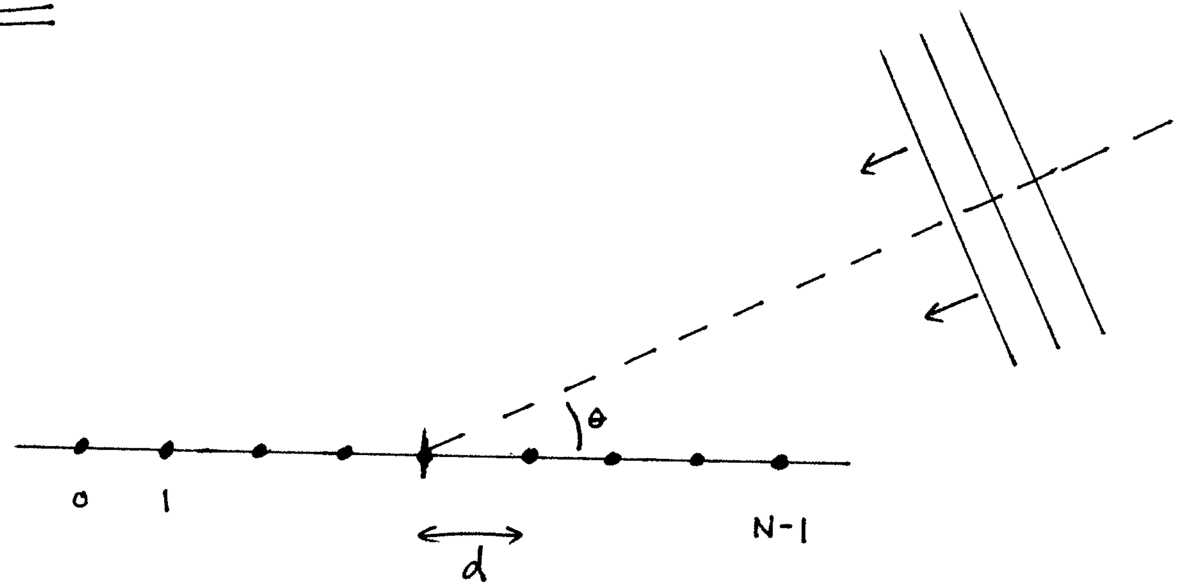
<u>far-field</u> $\Rightarrow$ plane waves at array



We will study the following scenario

- plane waves (far-field sources)
- linear array of equally spaced arrays
- sinusoidal signals
- white noise at sensors

The Picture



N sensors

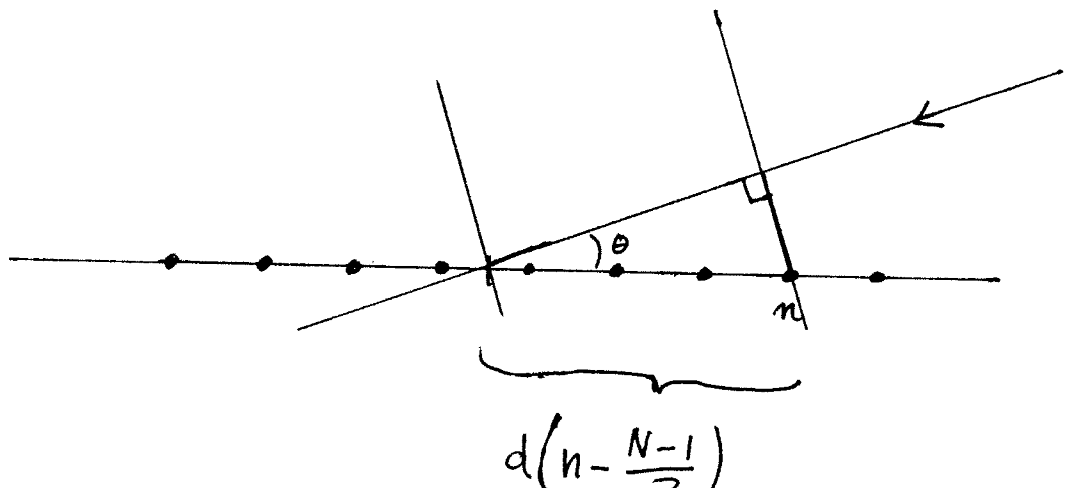
d = intersensor distance

origin = centroid of sensors (not a sensor location if N is even)

c = speed of plane wave

Exercise Suppose the signal is $s(t)$ at the origin. Express the signal at sensor number n in terms of d, c, n, and θ.

Solution



$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{d}{c}\left(n - \frac{N-1}{2}\right)\cos\theta$$

$$s^{(n)}(t) = s\left(t + \frac{d}{c}\left[n - \frac{N-1}{2}\right]\cos\theta\right)$$

Now assume $s(t)$ is a complex sinusoid:

$$s(t) = A \exp\{i(2\pi f_c t + \phi_c)\}$$

where

f_c = carrier frequency

ϕ_c = carrier phase

By the previous exercise, at the n^{th} sensor,

$$s^{(n)}(t) = A\exp\left\{i\left(2\pi f_c t + 2\pi\left(\frac{f_c}{c}\right)d\left[n - \frac{N-1}{2}\right]\cos\theta + \phi_c\right)\right\}$$

$$= A\exp\left\{i\left(2\pi f_c t + 2\pi \frac{d}{\lambda}\left[n - \frac{N-1}{2}\right]\cos\theta + \phi_c\right)\right\}$$

$\boxed{\text{since } c = f_c \lambda}$

The Big Idea

View t as fixed and n as a variable. What kind of "spatial" signal do we observe?

$$s^{(t)}[n] = A \cdot \exp\left\{ i\left(2\pi f n + \phi\right) \right\}$$

where

$$f = \frac{d}{\lambda} \cos\theta$$

$$\phi = 2\pi f_c t + 2\pi \frac{d}{\lambda} \cdot \frac{N-1}{2} \cdot \cos\theta + \phi_c$$

<u>Conclusion</u>: The spatial signal is again a complex sinusoid.

Assuming complex WGN $z[n]$ corrupts the sensor measurement at sensor n, we observe

$$x[n] = A \exp\left\{ i\left(2\pi f n + \phi\right) \right\} + z[n]$$

$\Rightarrow$ we can use $x[0], x[1], \ldots, x[N-1]$ as our data to compute a frequency estimate $\hat{f}$ using the techniques we just finished studying

$$\Rightarrow \quad \hat{\theta} = \cos^{-1}\left[\frac{\lambda}{d} \hat{f} \right]$$

Multiple Signals

A major advantage of arrays of omnidirectional sensors over directional sensors is the ability to estimate the bearing of multiple signals at the same time.

Suppose there are p sinusoids, each with the same carrier frequency f_c.

Then our spatial signal is

$$x[n] = \sum_{j=1}^{p} A_j \exp\{i(2\pi f_j n + \phi_j)\} + z[n]$$

$\Rightarrow$ $\hat{f}_1, \ldots, \hat{f}_p$ using sinusoidal frequency estimation

$\Rightarrow$ $\hat{\theta}_j = \cos^{-1}\left[\frac{\lambda}{d} \hat{f}_j\right]$

Question: Why do we need to assume each sinusoid has the same carrier frequency?

Answer: Recall $\lambda = \frac{c}{f_c}$. If the carrier frequencies are different, we don't know what λ to use when relating $\hat{f}_j$ to $\hat{\theta}_j$.

High Resolution Frequency Estimation

We need it for 3 reasons:

- angles / bearings may be very close
- array size may be constrained by space or cost (small N)
- $f_j = \frac{d}{\lambda} \cos \theta_j \in \left[-\frac{d}{\lambda}, \frac{d}{\lambda}\right]$, so if $\frac{d}{\lambda}$ is small, the frequencies are very bunched together. (if λ is large, physical constraints may prevent d from being large enough)

Comments

- If $\frac{d}{\lambda} > \frac{1}{2}$, aliasing can occur.

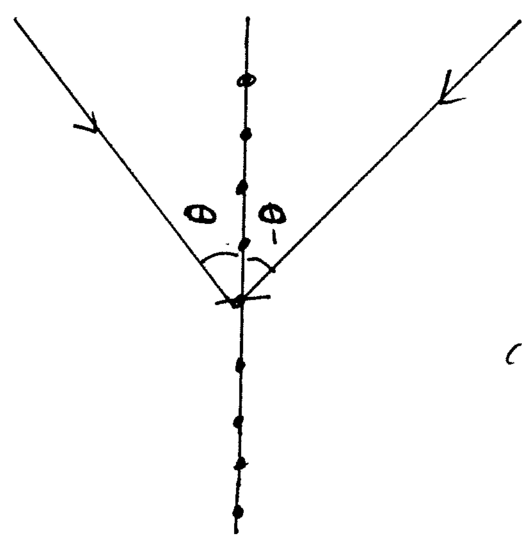
  In particular, $\theta \in [0, \pi)$

  $$\Rightarrow \quad f \in \left[-\frac{d}{\lambda}, \frac{d}{\lambda}\right]$$

  which is larger than $\left[-\frac{1}{2}, \frac{1}{2}\right]$

- Hence, $\frac{d}{\lambda} = \frac{1}{2}$ is ideal.

- A linear array cannot distinguish one side of the array from the other:



$$\cos\theta = \cos(-\theta)$$

$\Rightarrow$ need multiple arrays in such cases

Array Processing

... is a broad area

→ nonlinear arrays

→ nonuniform sensor spacing

→ location estimation

→ target tracking

→ 3D (we discussed 2D)

Spectral estimation bears on these problems too!

References:

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