Section 14

1. You are a top secret government operative seeking an organized crime lord. You know that he is either hiding in a warehouse or in a power plant. Every day you can only search one location - if you look in the warehouse and the crime lord is actually there, you will find him with probability 0.5. (depending on whether you are lucky that day). If you look in the power plant and the crime lord is actually there, you will find him with only probability 0.3. Prior to your search, you received a tip that the crime lord is in the power plant with probability 0.6.

(a) On the first day, should you search the warehouse or the power plant? Justify your answer with a calculation. (Hint: think about what you should calculate.)

(b) Suppose you search the warehouse in the first day but cannot find the crime lord. Given this information, would you now search the warehouse or would you switch to the power plant on the second day? Justify your answer with a calculation. (You can assume that the crime lord does not change locations from day to day.)

2. Your friend would like to play a game with you. He has two coins, a fair coin and a biased coin with probability $\frac{1}{3}$ of heads. He picks one of the two coins at random and flips it, showing you the result, and you have to guess which coin it is.

Let X be a random variable corresponding to the coin that your friend picks, with $X = 1$ if he picks the fair coin and $X = 2$ if he picks the biased coin. Let H_i be an indicator random variable such that $H_i = 1$ if the ith flip comes up heads.

(a) Are H_1 and H_2 independent? In other words, if you see a heads on the first flip, does that change the likelihood of seeing a heads on the second flip?

(b) Suppose you caught a glimpse of the coin your friend chose, so you know it was the biased coin. Given this information, are H_1 and H_2 independent?

(c) Suppose that your friend now picks the fair coin with probability q. If the first flip is heads, for what values of q would you guess that he picked the fair coin?
(Hint: Use the likelihood ratio $\frac{\Pr[H_1=1|X=1]}{\Pr[H_1=1|X=2]}$.)

3. Imagine that you have n drawers in your filing cabinet, and that you left your term paper in drawer k with probability D_k . Furthermore, suppose that these drawers are so messy, that even if you correctly guess that the term paper is in drawer j, the probability that you find it is p_j. What drawer should you start looking for your term paper in?

Now, suppose you search for your paper in a particular drawer, say drawer i, and the search is unsuccessful. What drawer should you search for next?