# Non-Iterative Reconstruction of Sparse Images from Limited Data

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# Presentation Overview

- Why sparse images and non-iterative?
- Review of "valid" 2-d deconvolution
- Valid reconstruction of sparse images [3]:

1. Valid deconvolution of bandlimited PSF;
2. Slightly underdetermined reconstruction;
3. Kronecker-product-based reconstruction.

- Valid phase retrieval of sparse images [1]
- Conclusion

# Presentation Overview

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- **Valid phase retrieval of sparse images [1]**
- **Conclusion**

# Why are sparse images important?

- DEF: 1-d x(n) is K-sparse if x(n) is nonzero only at K (unknown) values of index n.
- EX: x(n)={2,0,0,0,3,0,0,1,0,0,0,0,4,0,0,1,0} (all other values of x(n) are 0) is 5-sparse.
- Extension to 2-d (images) should be evident.
- Example: atoms in X-ray crystallography.
- Example: atoms in magnetic resonance force microscopy (MRFM).

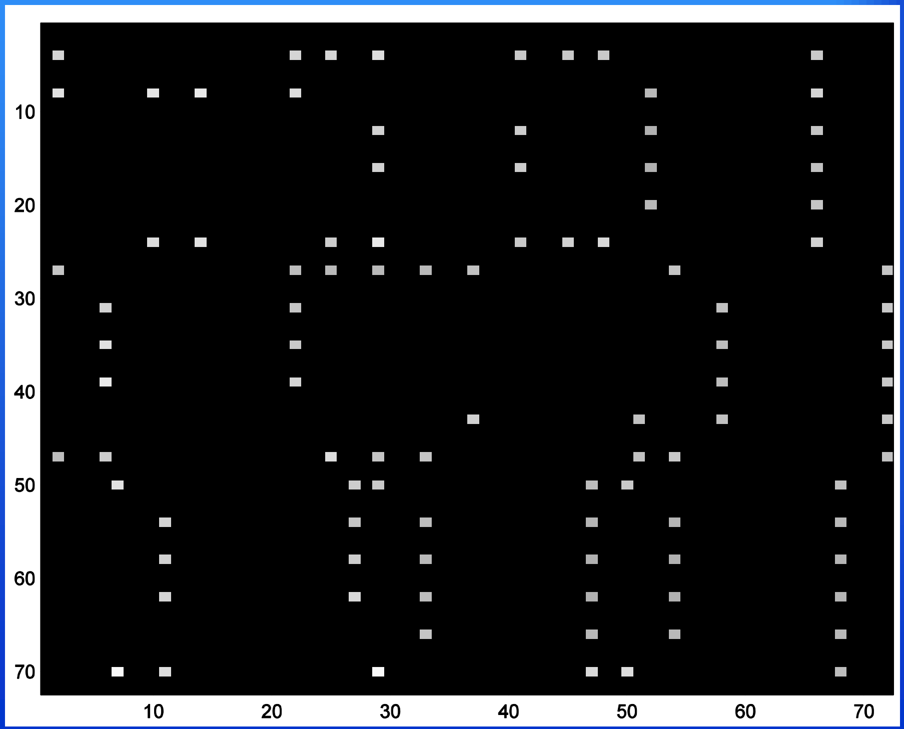
# What are sparsifiable images?

- x(n) is **sparsifiable** if $x(n)=\sum c(n,m)z(m)$ for some known matrix of basis functions c(n,m) and z(n) is sparse (x(n) sparse in some basis).
- Example: c(n,m) are wavelet basis functions.
- Extension to 2-d (images) is evident (but this requires 4 indices; more if wavelets are used!)
- Example: block letters or symbols (next slide).

# Example of sparsifiable image

Using corner detector, we can sparsify block letters:

Corner detector: $y(i,j)=x(i,j)-x(i,j-1)-x(i-1,j)+x(i-1,j-1)$

# Problems with iterative algorithms

- Does the algorithm converge at all?
- How long does it take to converge?
- To what does the algorithm converge?
- What bias is introduced by stopping?
- Can take long, unknown time to converge.
- Non-parallelizable in iteration number.
- Non-iterative: avoids all of these issues.

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# Complete 2-d convolution

$y(i,j)=\sum\sum h(m,n)x(i-m,j-n)$. **DFT**: $X(k)=\sum x(n)e^{-j2\pi nk/N}$ for k.

**Deconvolution**: $X(k_1,k_2)=Y(k_1,k_2)/H(k_1,k_2)$ for $|H(k_1,k_2)|>0$.

| | | | | |
|---|---|---|---|---|
| 1 | 4 | 1 | 5 | 9 |
| 2 | 6 | 5 | 3 | 5 |
| 8 | 9 | 7 | 9 | 3 |
| 2 | 3 | 8 | 4 | 6 |
| 2 | 6 | 4 | 3 | 3 |

x(i,j)

*

| | |
|---|---|
| 1 | 1 |
| 1 | 1 |

h(i,j)

=

| | | | | | |
|---|---|---|---|---|---|
| 1 | 5 | 5 | 6 | 14 | 9 |
| 3 | 13 | 16 | 14 | 22 | 14 |
| 10 | 25 | 27 | 24 | 20 | 8 |
| 10 | 22 | 27 | 28 | 22 | 9 |
| 4 | 13 | 21 | 19 | 16 | 9 |
| 2 | 8 | 10 | 7 | 6 | 3 |

y(i,j)=h(i,j)*x(i,j)

# Valid 2-d convolution

$y(i,j)=\sum\sum h(m,n)x(i-m,j-n)$. BUT: no image edge info used.

Deconvolution: Underdetermined—need info about image.

| | | | | |
|---|---|---|---|---|
| 1 | 4 | 1 | 5 | 9 |
| 2 | 6 | 5 | 3 | 5 |
| 8 | 9 | 7 | 9 | 3 |
| 2 | 3 | 8 | 4 | 6 |
| 2 | 6 | 4 | 3 | 3 |

*

| | |
|---|---|
| 1 | 1 |
| 1 | 1 |

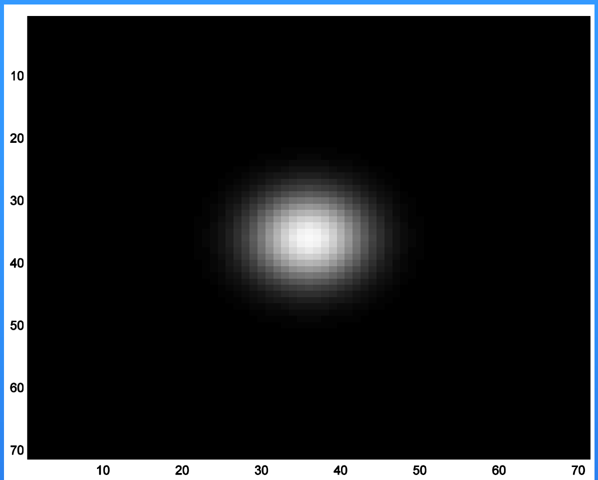
=

| | | | | | |
|---|---|---|---|---|---|
| * | * | * | * | * | * |
| * | 13 | 16 | 14 | 22 | * |
| * | 25 | 27 | 24 | 20 | * |
| * | 22 | 27 | 28 | 22 | * |
| * | 13 | 21 | 19 | 16 | * |
| * | * | * | * | * | * |

*** = unknown values**

# What 2-d convolution does to images


*

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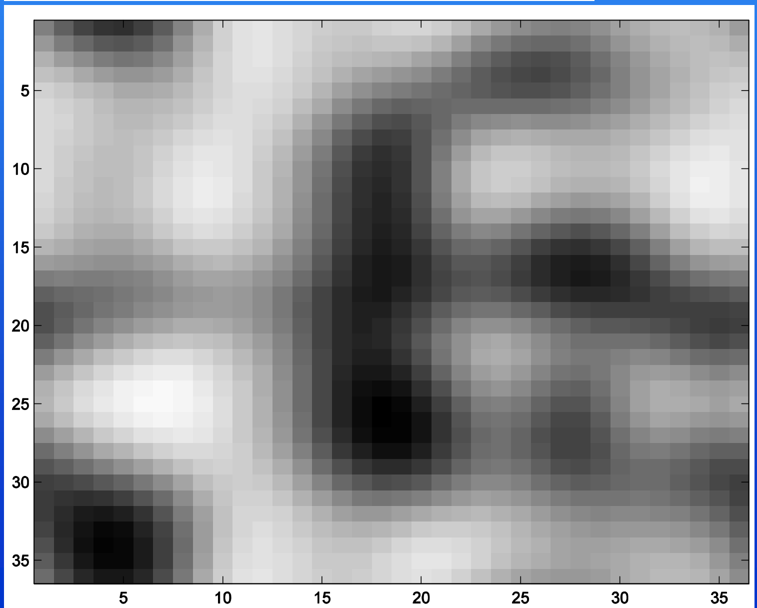


**valid deconvolution**

**also downsampled:**

**<u>Deconvolution</u>: undo**

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# Formulation of Basic Problem

- GIVEN: Underdetermined linear system.
- $y=Hx$. Data y: known M-vector.
- Solution x: unknown N-vector.
- Infinite number of solutions, since $M<N$.
- Compute the unique K-sparse solution x.
- Assume it exists (*a priori* knowledge).

# Why not use $\ell_1$ minimization?

- Minimize $\sum|x(n)|$ such that $y=Hx$ (constrained)
- Min $||y-Hx||^2 + \lambda\sum|x(n)|$ (LASSO functional)
- Min $||y-Hx||_1 + \lambda\sum|x(n)|$ (LAD functional)
- Min $||y-Hx||^2 + \lambda\sum|x(n)-x(n-1)|$ (total variation)

- Minimizing $\sum|x(n)|$ tends to sparsify $x(n)$ IF
- H is a random matrix (or other conditions)

# Why not use $\ell_1$ minimization?

- Minimize functionals using: gradient, or linear programming or coordinate descent (all iterative methods; may take long time)

- BUT: H matrix in image reconstruction is NOT a random matrix! $\ell_1$ doesn't work!

# Alternative to $\ell_1$ norm minimization

- Suppose x(n) has length=N and is K-sparse.
- Then there is an **indicator function** s(n) s.t.:
- s(n)x(n)=0 and DFT S(k) has length=K+1.

- DFT: $X(k)=\sum x(n)e^{-j2\pi nk/N}$ for N values of k.

- Locations of nonzero x(n): $\{n_1, n_2, n_3 \ldots n_K\}$.
- Polynomial $\sum S(k)z^k$ has K zeros at locations $\{\exp(-j2\pi n_1/N) \ldots \exp(-j2\pi n_K/N)\}$.

# Example: Indicator function

- x(n)={0,0,2,0,3,0,0,0}. Length=8; 2-sparse.
- s(n)={(1+j)/4, .177j, 0, .073, 0, -.177j, (1-j)/4, .427}
- S(k)={1, 1+j, j, 0, 0, 0, 0, 0}. Roots: {-j, -1}.

- x(n) K-sparse→s(n)x(n)=0→S(k)*X(k)=0.
- K+1 unknowns S(k) impose sparsity on x(n).
- Use this in the following **NEW** algorithms.

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1. Valid deconvolution of bandlimited PSF;
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- Valid phase retrieval of sparse images [1]
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# 1. Bandlimited matrix H: Needs

- ASSUME: Each row of H bandlimited to M.
- THEN: K-sparse x(n) computed by solving:
- (1) M×M system to compute X(k),$0 \leq k \leq M/2$;
- (2) K×K Toeplitz to compute s(n), locations $n_i$
- (3) K×K system to compute x(n) values.

- APPLICATION: Deconvolving bandlimited point-spread functions (PSF) h(i,j) from x(i,j).
- EXAMPLE: 2-d Gaussian PSF.

# 1. Bandlimited matrix H: Procedure

- Let $H(i,k)=\sum h(i,n)e^{-j2\pi nk/N}$=DFT{rows of h(i,j)}
- $y(i)=\sum h(i,j)x(j)=\sum H(i,k)^{*}X(k)/N$ (Parseval).
- H(i,k) rows bandlimited to M implies H(i,k)=0 for $M/2<k<N-M/2$ for each row #i of H(i,k).

- Solve $M\times M$ linear system for X(k), $0\le k\le M/2$.
- Solve $K\times K$ Toeplitz equations $S(k)*X(k)=0$.
- Solve $K\times K$ linear system for values of x(n).

# 1. Bandlimited matrix H: Example

- **IMAGE** x(i,j): 72×72 and sparsifiable.
- **PSF**: 2-d Gaussian $h(i,j)=0.98^{(i^2+j^2)}$ bandlimited
- **DATA**: y(i,j)=h(i,j)*x(i,j) & downsampled, since h(i,j) bandlimited implies y(i,j) also bandlimited.

- **GOAL**: Compute x(i,j) from downsampled y(i,j).
- **NOTE**: Clearly underdetermined linear problem (see next slide for numerical details).

# 1. Bandlimited matrix H: Example

- Unknowns: $72^2=5184$ pixels x(i,j).
- Knowns: $36^2=1296$ values y(i,j) of downsampled h(i,j)*x(i,j) (cyclic *).

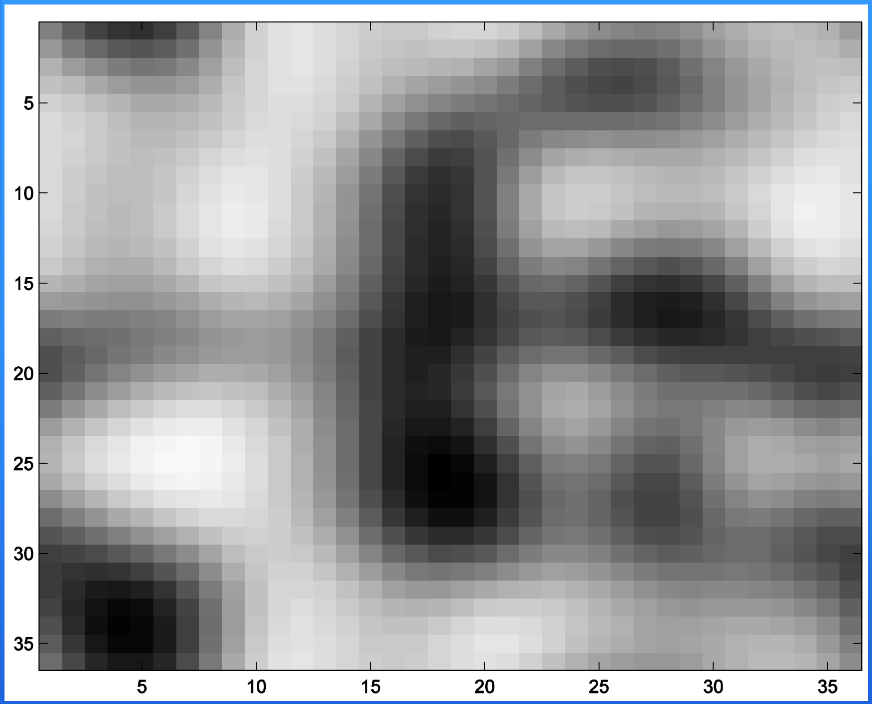
- Side information: x(i,j) sparsifiable by $z(i,j)=x(i,j)-x(i,j-1)-x(i-1,j)+x(i-1,j-1)$.
- y(i,j) known at 19×19 lowest wavenumbers.
- NEED: sparsified z(i,j) is $10^2-1=99$-sparse.

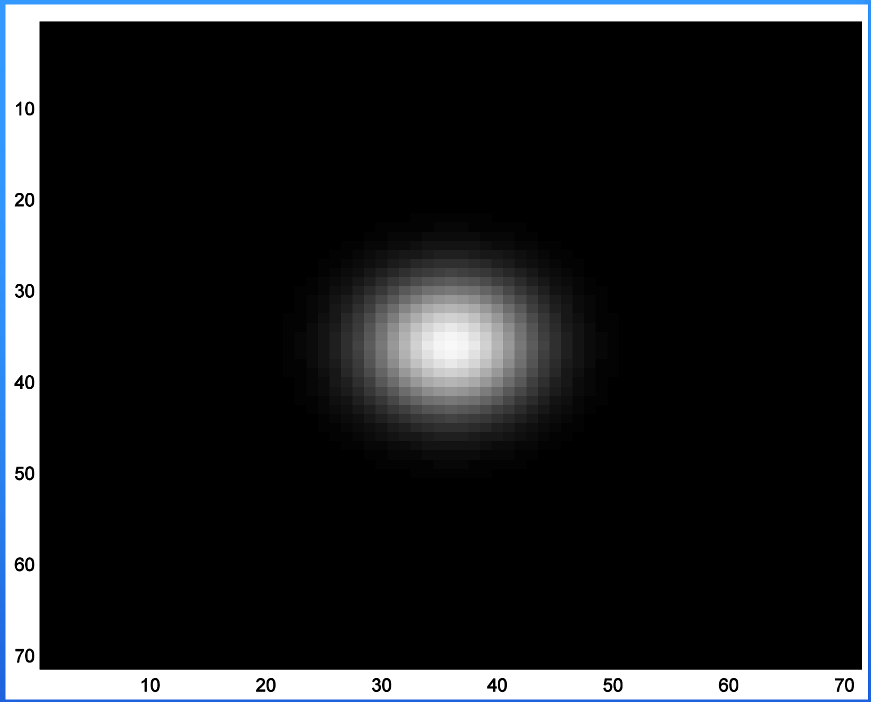
# 1. Bandlimited matrix H: Example

- Computational requirements:
- Null of 100×100 Toeplitz-block-Toeplitz;
- 72×72 2-d DFT of 10×10 rearrangement of null vector of Toeplitz-block-Toeplitz;

- Solution of 98×98 to compute z(i,j) values;
- Deconvolve corner detector: z(i,j)→x(i,j). Requires knowledge of 2 edges of x(i,j).

# 1. Bandlimited matrix H: Example



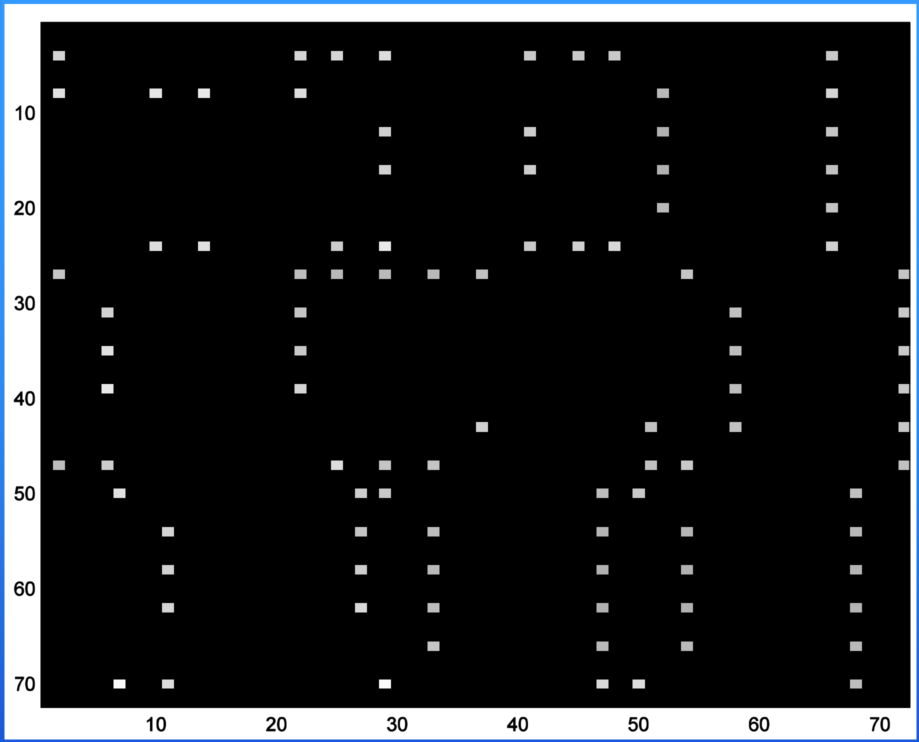




**Blurred and downsampled image data.**

**2-d Gaussian blurring PSF h(i,j)**

**Can you guess the original image?**

# 1. Bandlimited matrix H: Example





Reconstructed sparsified image z(i,j)

Reconstructed original image x(i,j)

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# 2. Slightly Underdetermined H: Needs

- **ASSUME**: y=Hx only *slightly underdetermined*:
- N>(N-M)(K)=(#underdetermined)(#nonzero x(n))
- Actually need N>(N-M+1)(K+1) (counting issues).

- **APPLICATION**: Valid deconvolution of PSFs that are spatially varying; other underdetermined linear transformations of K-sparse signals.

# 2. Slightly Underdetermined H: Procedure

- **THEN**: $y=Hx \rightarrow [H\ {-y}][x^T\ 1]^T=0$ (include y in H).
- Rename: $H=[H\ {-y}]$ and $x^T=[x^T\ 1]^T$ in the sequel.

- Now $y=Hx$ has become $0=Hx$. Using Parseval:
- $0=Hx=\sum h(i,j)x(j)=\sum H(i,k)^{*}X(k)=\underline{Hx}$ (DFT of H,x).

- $\underline{x}=\underline{Gw}$ where $\underline{G}$ spans right nullspace of $\underline{H}$.

# 2. Slightly Underdetermined H: Procedure

- **BUT**: **G** and vector **w** have dimensions N-M.

- **SO**: $S(k)^* \sum G(i,k) w(k)=0$ is N equations in (N-M) unknowns **w**(k) and K unknowns S(k). Becomes:

N **linear** equations in (N-M)(K) unknowns $S(k_1)w(k_2)$

# 2. Slightly Underdetermined H: Example

- IMAGE x(i,j): 30×30; sparsifiable to 12-sparse.
- LINEAR TRANSFORMATION H: Random 832×900 matrix times inverse corner detector.
- DATA: y=Hx where x(i,j) unwrapped by rows.

- GOAL: Compute x from y. Underdetermined.
- NEED: N>(N-M+1)(K+1) not (N-M)K (counting)
- HAVE: 900>897=(900-832+1)(12+1) so can do it.

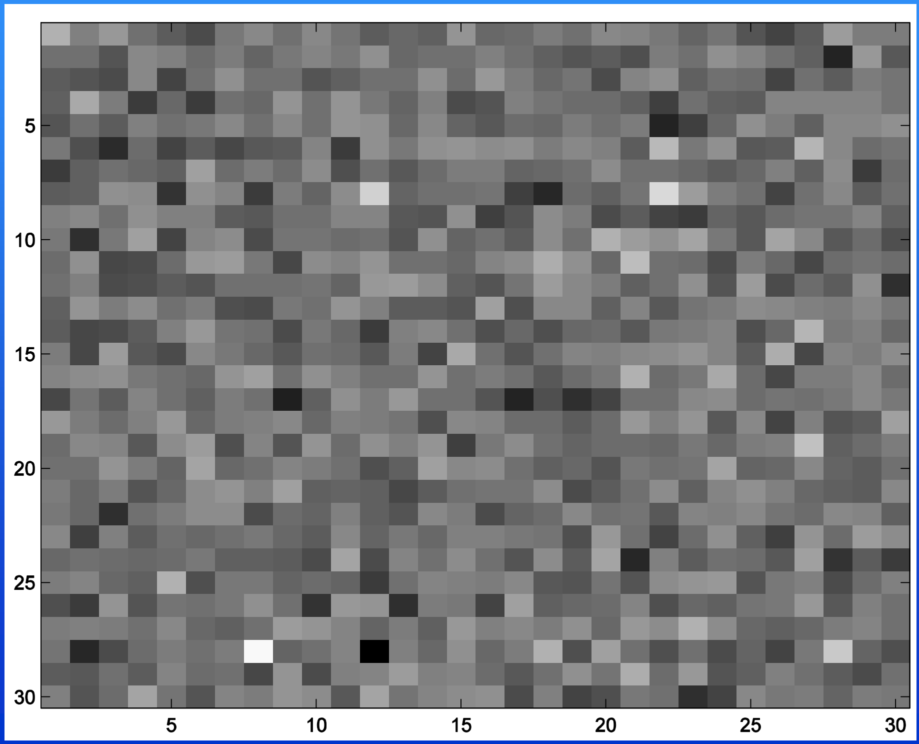
# 2. Slightly Underdetermined H: Example

- **Computational requirements:**
- **Null of 900×897 Toeplitz-blocks matrix;**

- **Rearrange null vector into 69×13 matrix;**
- **Rank-one factorization of this matrix;**
- **900-point DFT of length=13 rank-one factor;**
- **12 values of this were zero; these specified locations of nonzero elements of sparsified x.**

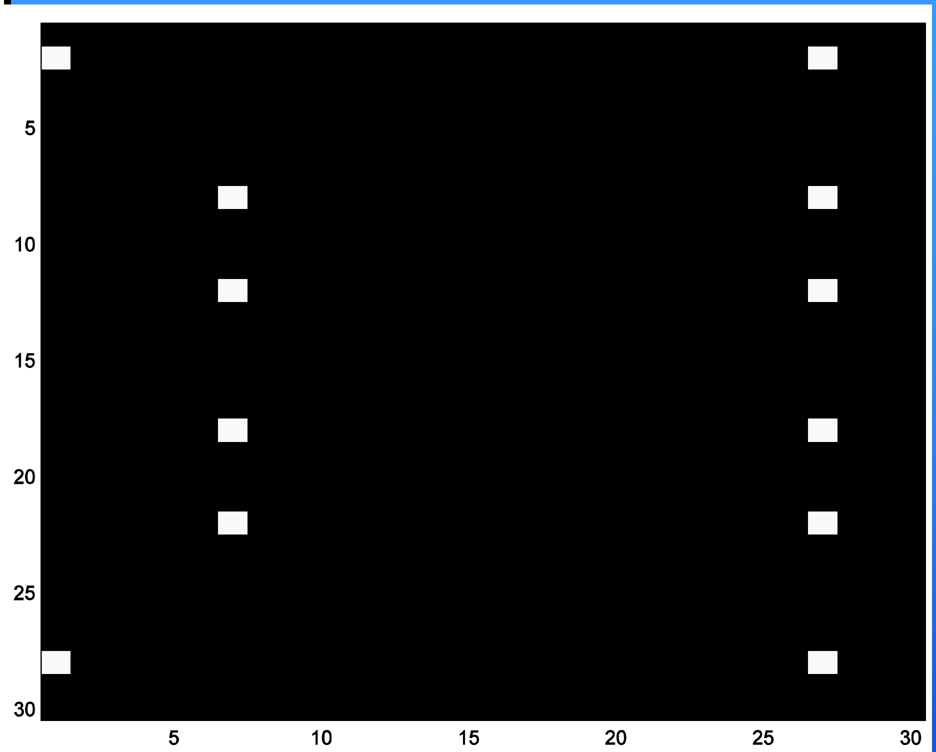
# 2. Slightly Underdetermined H: Example

**832×900 is only *slightly* underdetermined linear system.**
**Can't we just use least-squares to find 12 nonzero values?**
**This is the least-squares solution. Find 12 nonzero values:**

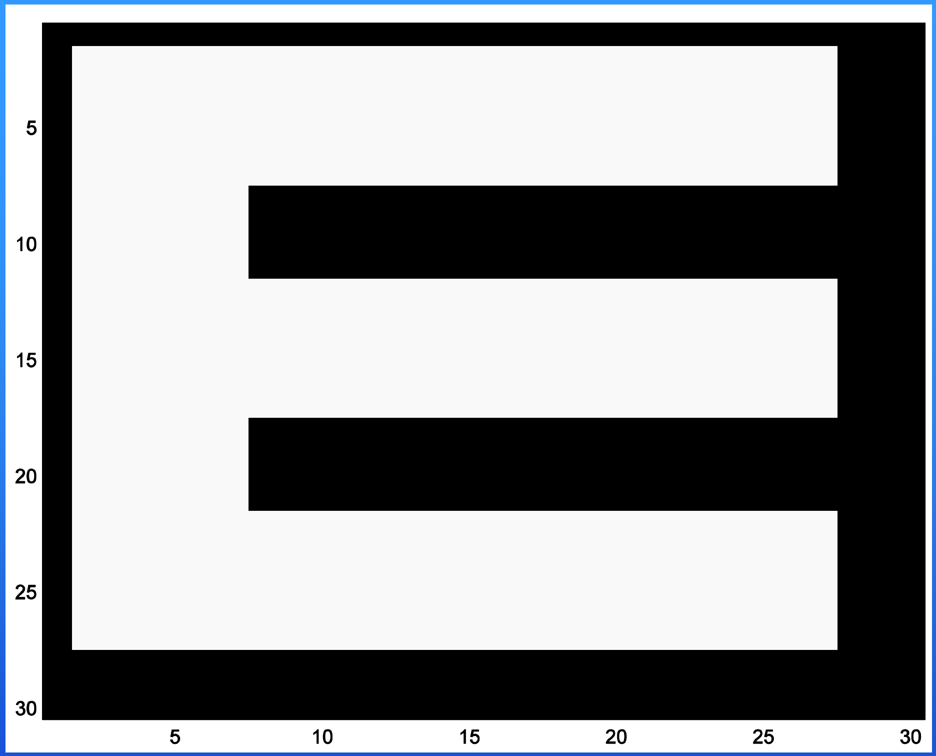



**This is the least-squares SOLUTION, not the data!**
**Only 12 of these pixels are supposed to be nonzero!**
**Can you pick out the 12?**
**HINT: They aren't the brightest pixels you see.**

# 2. Slightly underdetermined H: Example




Reconstructed sparsified image z(i,j)




Reconstructed original image x(i,j)

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# <u>3. Kronecker Product H: Needs</u>

- <u>GIVEN</u>: $y=Hx$ where $H=H_1\times H_2$
- $H_1\times H_2$ = Kronecker product of $H_1$ & $H_2$.

- y is an $M^2$ vector & x is an $N^2$ vector.
- x is (M-1) sparse or less; *very* sparse.

- <u>GOAL</u>: Compute x from y. Note $M^2 \ll N^2$ .
- <u>ADVANTAGE</u>: *Much* less computation.

# 3. Kronecker Product H: Review

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 5 & 6 & 10 & 12 \\ 7 & 8 & 14 & 16 \\ 15 & 18 & 20 & 24 \\ 21 & 24 & 28 & 32 \end{bmatrix}$$

Relevant properties of the Kronecker product here:

$(A \times B)(C \times D) = (AC) \times (BD).$

$\mathrm{vec}(AXB) = (B^T \times A)\mathrm{vec}(X).$

$\mathrm{vec}(Y) = (H_1 \times H_2)\mathrm{vec}(X)$ means $Y = H_2 X H_1^T.$

$$\mathrm{vec}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \\ 4 \end{bmatrix}$$

# 3. Kronecker Product H: Applications

- **2-d deconvolution** with **separable 2-d PSF**.
  Example: 2-d Gaussian PSF is separable.

- 2-d reconstruction from **partial DFT** data.
  2-d DFT is Kronecker product of 1-d DFTs.

- Image sparsifiable by **separable 2-d transform**.
  2-d wavelet transform is usually separable.

# 3. Kronecker Product H: Procedure [1/3]

- $y=(H_1 \times H_2)x$ same as $Y= H_2 X H_1^T$ where:
- $vec(X)=x$ and $vec(Y)=y$. $X$ is N×N; $Y$ is M×M.
- SVD's: $H_1=U_1 S_1 V_1$ and $H_2=U_2 S_2 V_2$ (identical?)

- $Y= H_2 X H_1^T=(U_2 S_2 V_2)X(U_1 S_1 V_1)^T$ becomes
- $V_2 X V_1^T=(S_2)^{-1} U_2^T Y U_1 (S_1)^{-1}$ computed from y.

# 3. Kronecker Product H: Procedure [2/3]

- $V_2XV_1^T$ is M×N but has rank at most M-1, since at most M-1 entries of X are nonzero.
- Can have more than M-1 nonzero entries of X if some lie on same row or column: Need at most M-1 nonzero-containing rows and M-1 columns.
- Null n of $V_2XV_1^T$ is same as null of $XV_1^T$ .

# 3. Kronecker Product H: Procedure [3/3]

- $i^{th}$ row of X all zeros→$i^{th}$ element of $XV_1^Tn=0$.
- $(i,j)^{th}$ element of X nonzero→($j^{th}$ row of $V_1^T$)$n=0$.
- Zeros of $V_1^Tn$→X columns with nonzero element.
- Repeat with $(V_2XV_1^T)^T$→X rows with nonzeros.

# 3. Kronecker Product H: Example

- $256 \times 256$ sparse image X with 21 nonzero pixels.
- $22^2 = 484 \times 65536 = 256^2$ random system matrix H.
- H=Kronecker product of two $22 \times 256$ matrices.

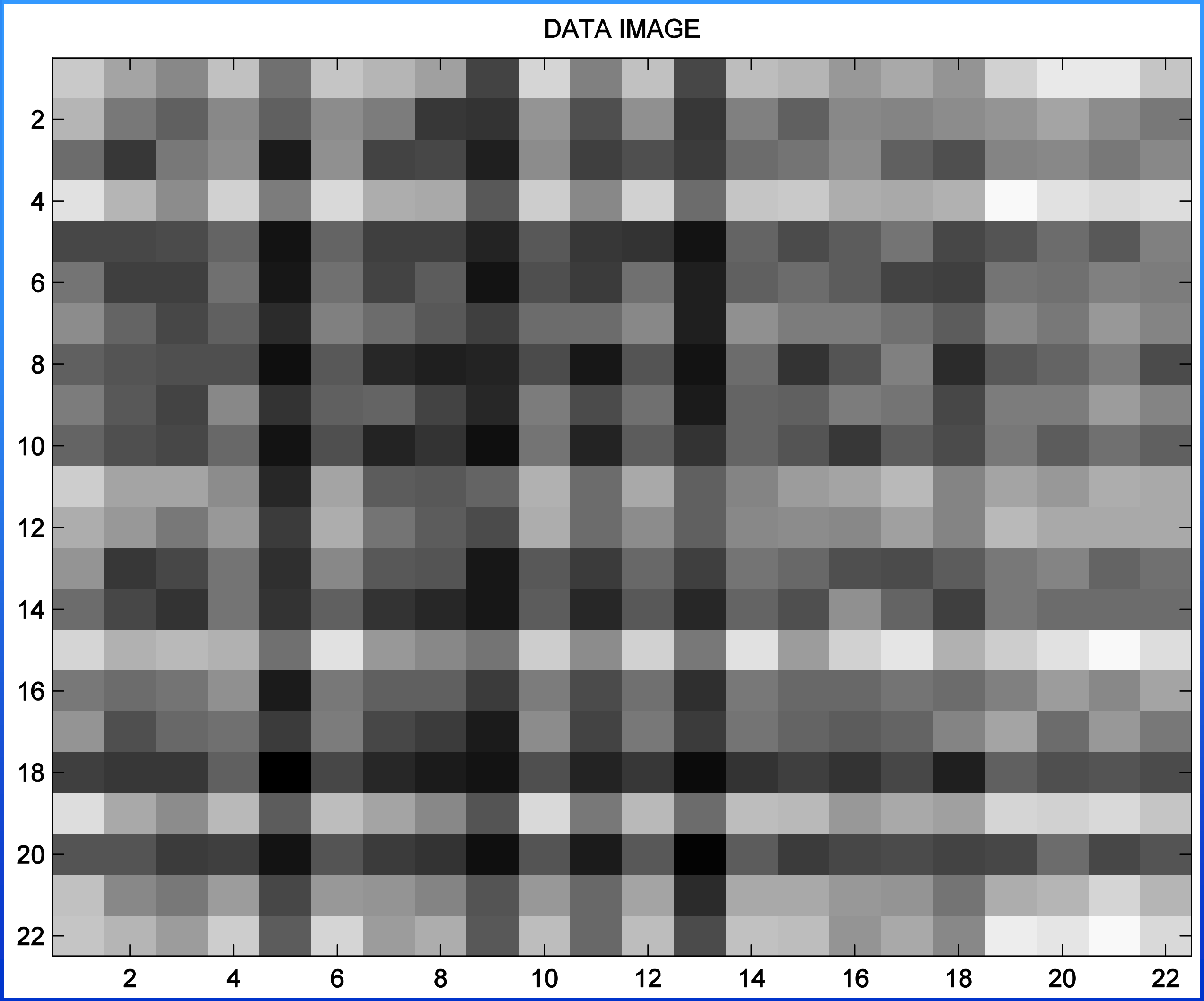
- Goal: Compute unknown 65536-element x from known 484-element y. Know that x is 21-sparse.

# 3. Kronecker Product H: Example

- **Computational requirements:**
- **SVD's of two 22×256 matrices (maybe identical). Can precompute these for given imaging system.**

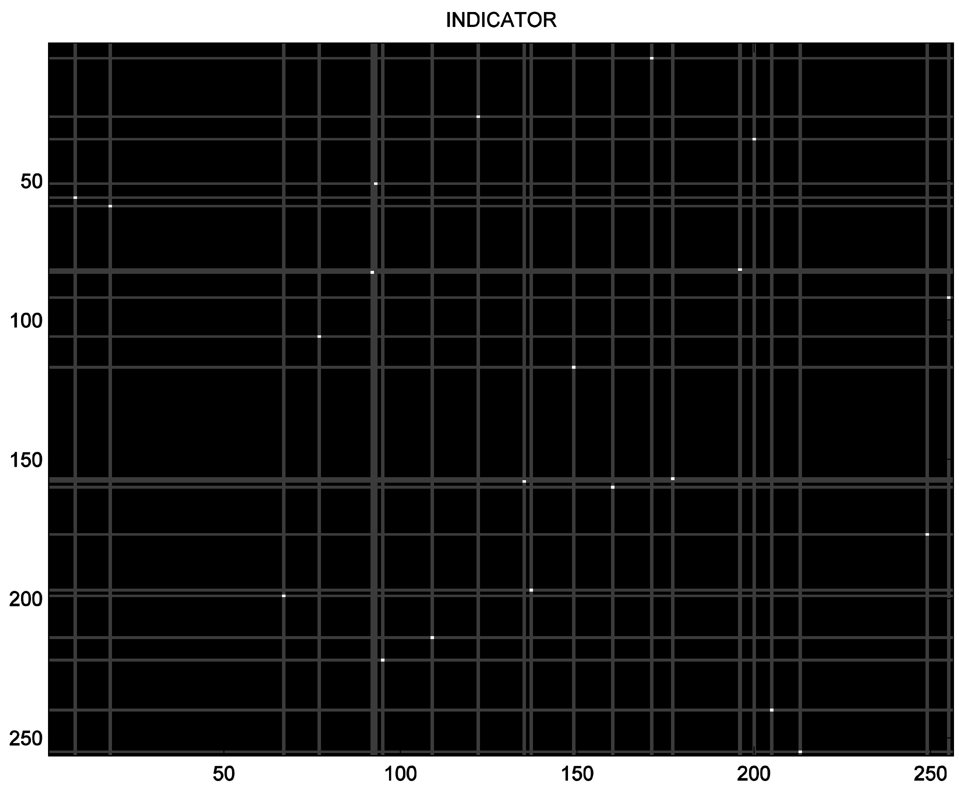
- **Left and right nulls of 22×22 data matrix.**
- **Compute $V_1^T n$ from null n; repeat for $V_2^T n$.**
- ***Very* little computation for this big a problem!**

# 3. Kronecker Product H: Example

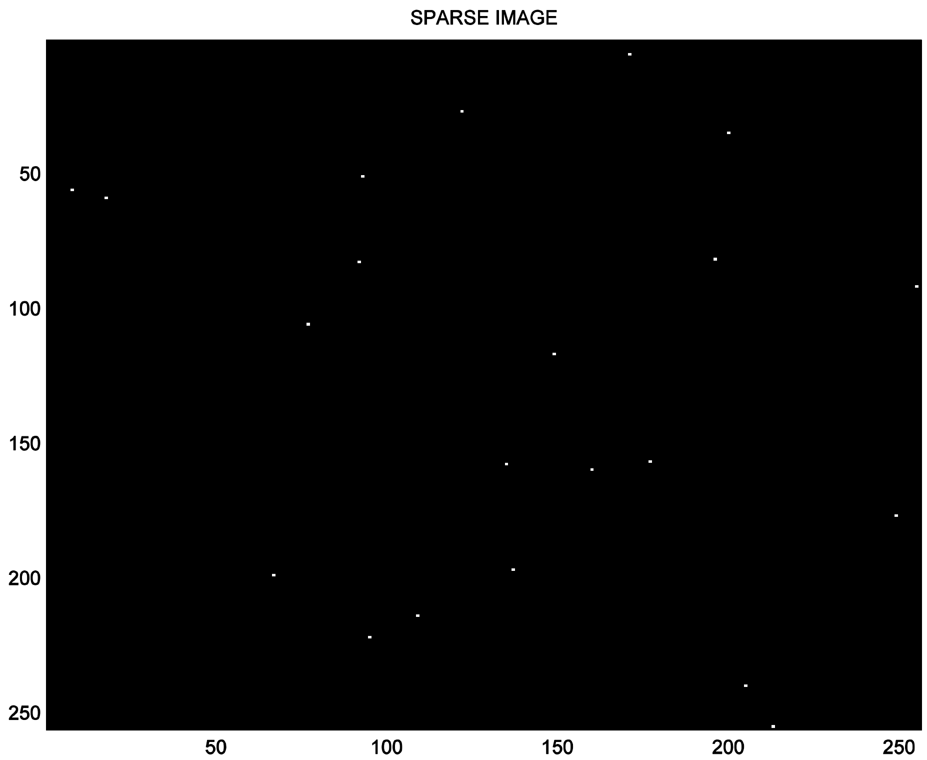



**Original data arranged into a 22 by 22 array. Can you guess locations of 21 nonzero pixels?**

# 3. Kronecker Product H: Example




Locations of possible nonzero pixels




Original image with 21 nonzero pixels

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# 4. Phase Retrieval of Sparse Images

- **GIVEN**: 2-d DFT Fourier magnitude $|X(k_1,k_2)|$.
- **BUT**: ***Don't*** **have Fourier *phase*** $\arg\{X(k_1,k_2)\}$.
- **GOAL**: Compute $x(i,j)$ from 2-d DFT $|X(k_1,k_2)|$.
- **HENCE**: "Phase retrieval" (from magnitude).
- **APPLICATIONS**: X-ray crystallography; optics (measure only diffraction patterns); astronomy.

# 4. Phase Retrieval: Problem set-up

- **ASSUME**: x(i,j) is either *sparse* or *sparsifiable* by an LTI transformation (e.g., corner detector).
- **GIVEN**: Autocorrelation $y(i,j)=DFT^{-1}\{|X(k_1,k_2)|^2\}$
- **UNWRAP**: 2-d problem to 1-d using either:
- Kronecker transformation: substitute $y=x^M$ in:

$$Y(x,y)=X(x,y)X(x^{-1},y^{-1}) \rightarrow Y(x,x^M)=X(x,x^M)X(x^{-1},x^{-M});$$

- Agarwal-Cooley convolution (residue # system).

# 4. Phase retrieval: Ambiguities

- SCALE FACTOR: Solution x(n)→-x(n) also.
- TRANSLATION: Solution x(n)→x(n-d) also.
- REVERAL: Solution x(n)→x(-n) also a solution.
- These extend to 2-d case in obvious fashion.

- All of these will appear in the sparse algorithm!

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# 4. Phase Retrieval: Problem set-up

- **GOAL**: Solve 1-d phase retrieval; rewrap to 2-d.
- **SOLVE**: $y(n)=y(-n)=x(n)*x(-n)=\sum x(i)x(i+n)$.
- If $x(n)$ sparsifiable to $z(n)$ by $x(n)=h(n)*z(n)$ for some known function $h(n)$ (e.g., corner detector):
- $y(n)=[h(n)*h(-n)]*[z(n)*z(-n)]$. Then deconvolve $h(n)*h(-n)$ from $y(n)\rightarrow$sparse problem $z(n)*z(-n)$.

# 4. Phase Retrieval: Algorithm [1/5]

- **ASSUME**: Each y(n) is a *single* x(i)x(i+n) term. True if x(n) is sparse and sampling of x(n) fine.
- **THEN**: Can replace nonzero x(n) and y(n) with 1 to find locations of nonzero x(n). Then actual x(n) computed from rank-one decomposition of r(n).

- **GIVEN**: r(n)=r(-n) support $-M \leq n \leq M$ for some M.
- **Initialize**: x(0)=x(M)=1 since r(M)=1.
- **NOTE**: This resolves *translation ambiguity*!

# 4. Phase Retrieval: Algorithm [2/5]

- <u>Recursion#1</u>: Let $n_1$ be next *largest* n s.t. $r(n) \neq 0$.
- Either $x(n_1)$ or $x(M-n_1) \neq 0$, but which one?
- Can't tell at this point-this is *reversal ambiguity*!

- Pick, without loss of generality, $x(n_1) \neq 0$.

# 4. Phase Retrieval: Algorithm [3/5]

- **Recursion #2**: Let $n_2$ be next *largest* n s.t. $r(n)\neq 0$.
- Either $x(n_2)$ or $x(M-n_2)\neq 0$, but which one?
- Now *can* tell! Check the following two cases:

- If $x(n_2)\neq 0$, then $r(n_1-n_2)\neq 0$ and $r(|M-n_1-n_2|)=0$.
- If $x(M-n_2)\neq 0$, then $r(|M-n_1-n_2|)\neq 0$ and $r(n_1-n_2)=0$.
- This specifies which of $x(n_2)$ or $x(M-n_2)\neq 0$.

# 4. Phase Retrieval: Algorithm [4/5]

- Recursion #3: Let $n_3$ be next *largest* n s.t. $r(n)\neq 0$.
- Either $x(n_3)$ or $x(M-n_3)\neq 0$, but which one?
- Suppose $x(n_2)$, not $x(M-n_2)$, was $\neq 0$. Then:

- If $x(n_3)\neq 0$, then $r(n_1-n_3)\neq 0$ and $r(n_2-n_3)\neq 0$.
- If $x(M-n_3)\neq 0$, $r(|M-n_3-n_1|)\neq 0$ and $r(M-n_3-n_2)=0$.
- This specifies which of $x(n_3)$ or $x(M-n_3)\neq 0$.

- NOTE: As recursions progress, more checks.

# 4. Phase Retrieval: Algorithm [5/5]

- **AT END**: Have all indices $n_j$ at which $x(n_j) \neq 0$.
- **THEN**: Each $r(n_i)=x(n_j)x(n_j+n_i)$ for a *known* $n_j$.

- **SO**: Form symmetric matrix of nonzero $r(n_i)$. Rank-one factorization→actual $x(n_j)$ values.

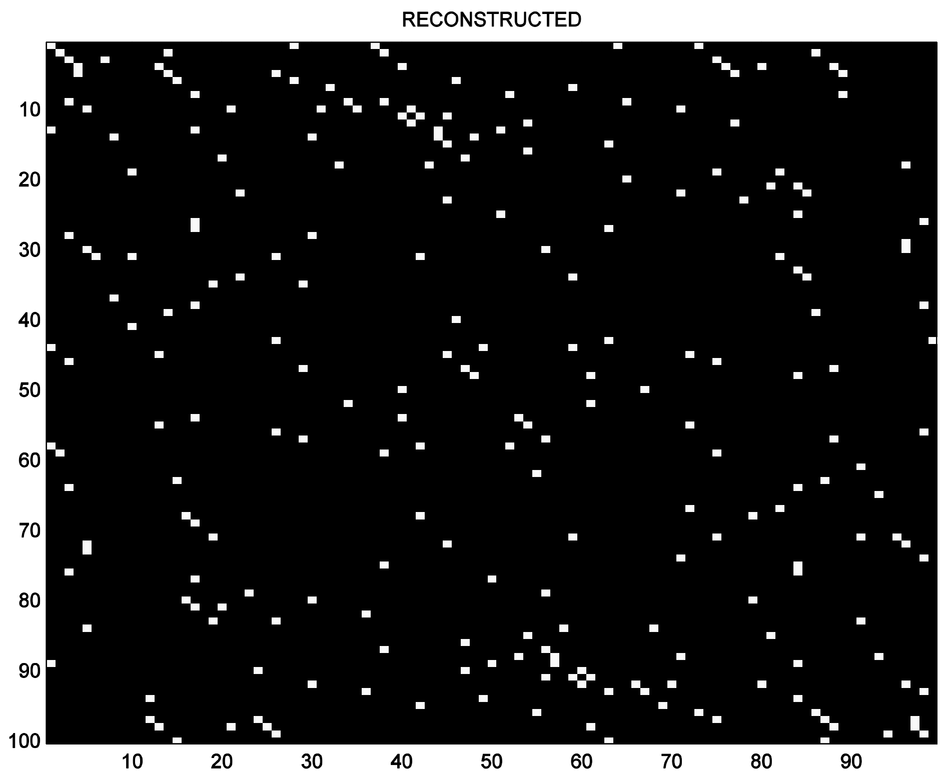
- **BUT**: Sign ambiguity in outer product: This is *Scale factor ambiguity*!

# <u>4. Phase Retrieval: Example #1</u>

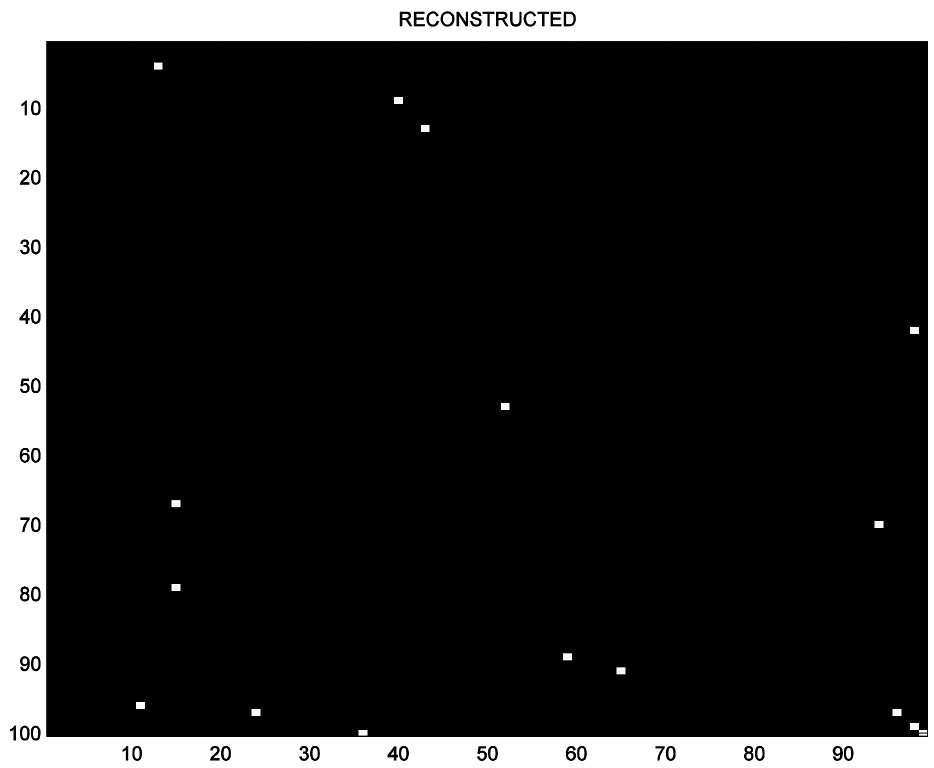
- **Sparse 100×99 image; 16 nonzero pixels.**
- **<u>GIVEN</u>: 100×99 *cyclic* autocorrelation; no image support constraint; just sparse.**
- **<u>GOAL</u>: reconstruct sparse 100×99 image.**

- **<u>NOTE</u>: Agarwal-Cooley used to map to 1-d.**
- **<u>NOTE</u>: Cyclic autocorrelation is 240-sparse; 16 values $x(n)\neq 0 \rightarrow 16(16\text{-}1)=240$ values $r(n)\neq 0$.**

# 4. Phase Retrieval: Example #1




Autocorrelation (zeroth lag suppressed)



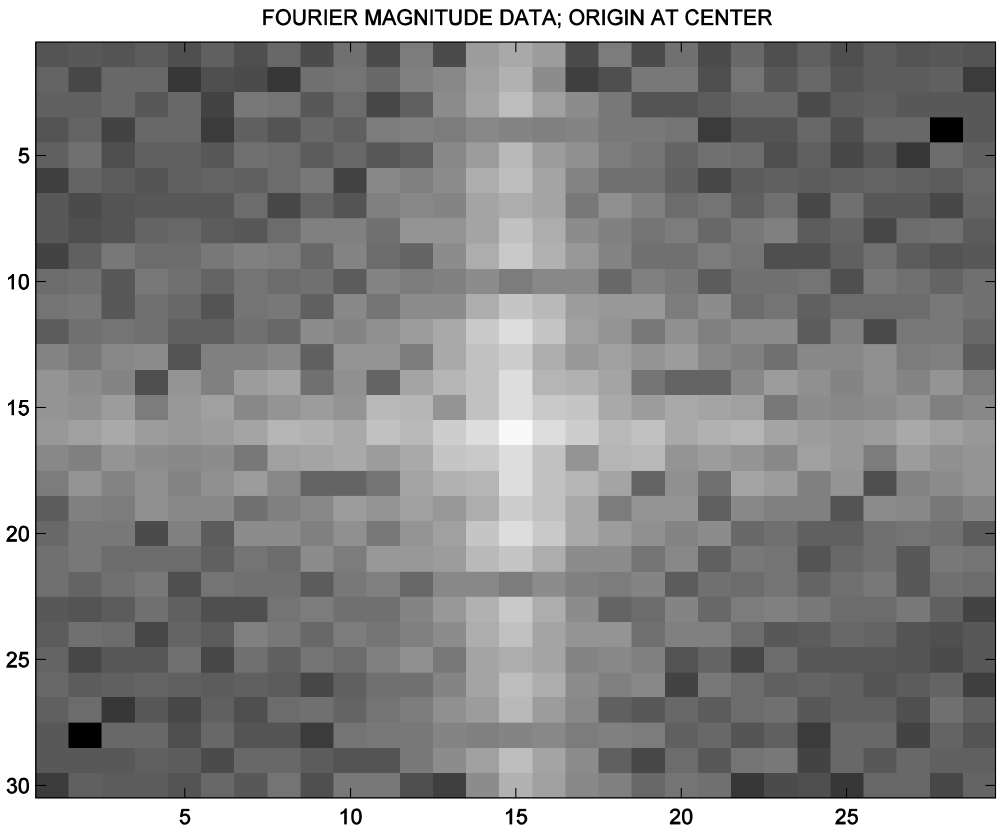

Reconstructed 16-sparse image

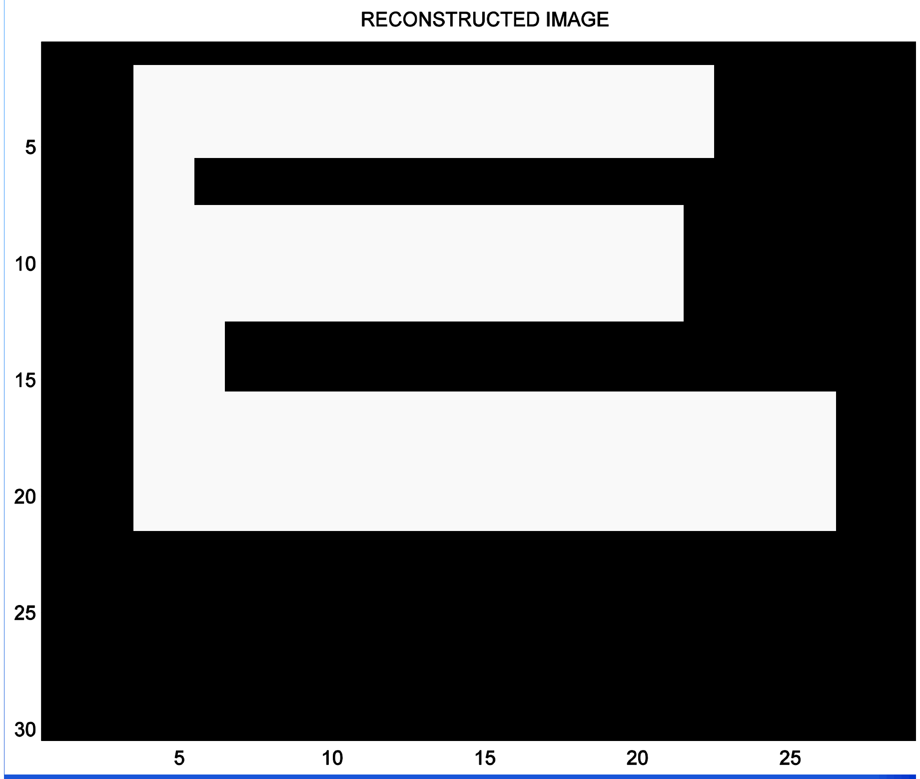
# 4. Phase Retrieval: Example #2

- **Sparsifiable** (by corner detector) 30×29 image.
- **GIVEN**: 30×29 *cyclic* autocorrelation of image; no image support constraint; just sparsifiable.
- **GOAL**: reconstruct **sparsifiable** 30×29 image.

- **NOTE**: Agarwal-Cooley used to map to 1-d.
- **NOTE**: $1^{st}$ deconvolve the corner detector from cyclic autocorrelation; then it is 132-sparse: 12 values $x(n) \neq 0 \rightarrow 12(12-1)=132$ values $r(n) \neq 0$.

# 4. Phase Retrieval: Example #2




**Fourier transform magnitude**

**Reconstructed sparsifiable image**

**note no support constraint known**

# 4. Phase Retrieval: Example #2

- NOTE: Weird-looking block letter "E." Why? Need to ensure that after deconvolving corner detector: Each y(n) is a single x(i)x(i+n) term.
- NOTE: In a realistic-size problem, this is not likely to be an issue (use fine discretization).
- Used small-size problem to illustrate the issue.
- NOTE: Do need a small support constraint: 2 edges of image are row and column of zeros, so can deconvolve corner detector from image.

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# CONCLUSION

- Non-iterative algorithms are fast: Most of these require only solution of an M×M linear system.
- 1st: For bandlimited valid image deconvolution
- 2nd: For non-bandlimited valid deconvolution with non-separable PSF; valid linear transform
- 3rd: For separable valid linear transforms of very sparse or sparsifiable images; VERY fast.
- Phase retrieval of sparse or sparsifiable images

# THANK YOU FOR LISTENING!

- **Papers and Matlab code for small examples at: http://www.eecs.umich.edu/~aey/sparse.html**
- **I would like to thank Jison for his hospitality (and for being such a good Ph.D student!)**
- **Any questions?**