PRINT YOUR NAME HERE:

HONOR CODE PLEDGE: "I have neither given nor received aid on this exam, nor have I concealed any violations of the honor code." Open book; SHOW ALL OF YOUR WORK!

SIGN YOUR NAME HERE:

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(25) 1. We observe $r(t) = \begin{cases} A\sin(3\pi t) + B\sin(5\pi t) + n(t) & \text{under } H_1 \\ C\sin(3\pi t) + n(t) & \text{under } H_0 \end{cases}$ on interval $0 \leq t \leq 2$.
$n(t)$ is 0-mean WGN with $S_n(\omega) = 1$. A, B, C are constants (see below).

(5) a. Choose 2 orthonormal basis functions and draw a signal space diagram.
(5) b. Write down the optimal detector for deciding H_0 vs. H_1 from $\{R(t), 0 \leq t \leq 2\}$.
(5) c. If $A = B = 3$ and $C = 1$, compute d^2 for this detector.
(5) d. If $A = B = 3$, what value of C gives the *worst* performance, in terms of d^2?
(5) e. Prove that increasing $|B|$ increases d^2, and that this is *not* true for $|A|$.

WRITE ANSWERS HERE:

(a): $\phi_1(t) =$

$\phi_2(t) =$

(b):

(c): $d^2 =$

(d): $C =$

(e):

(35) 2. We observe $r(t) = \begin{cases} At^2 + w(t) & \text{under } H_1 \\ w(t) & \text{under } H_0 \end{cases}$ over interval $-1 \leq t \leq 1$

$w(t)$ is 0-mean Gaussian with $S_w(\omega) = 10^6/[(\omega^2 + 4000)(\omega^2 + 9000)]$.

(5) a. Write down the Karhunen-Loeve expansion of $w(t)$ over $-1 \leq t \leq 1$. Give *explicit* expressions for $\phi_n(t)$ and λ_n.

(5) b. Given an arbitrary function $f(t)$, write down an expression for $\int Q(t,s)f(s)ds$.

(10) c. Write down the optimal detector for deciding H_0 vs. H_1 from $\{R(t), -1 \leq t \leq 1\}$. Let $A = 3$ and simplify your answer as much as possible.

(5) d. Let $A = 3$ and compute d^2 for this detector.

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(10) e. Now we *know* H_1 is true, but A is now an unknown constant. Compute $\hat{A}_{MLE}(\{R(t), -1 \leq t \leq 1\})$. Simplify as much as possible.

WRITE ANSWERS HERE:

(a): $\phi_n(t) =$

$\lambda_n =$

(b): $\int Q(t,s)f(s)ds =$

(c):

(d): $d^2 =$

(e): $\hat{A}_{MLE} =$

#1:

#2:

#3:

$\sum$:

(40) 3. We observe $r(t) = \begin{cases} A\sqrt{2}\sin(m\pi t) + w(t) & \text{under } H_1 \\ w(t) & \text{under } H_0 \end{cases}$ over interval $0 \le t \le 1$.

$w(t)$ is a Wiener process obtained from integrating $n(t)$ from Problem #1.

For parts (a)-(c): $A = 3$ and $m = 5.5$.

(5) a. Write down the Karhunen-Loeve expansion of $w(t)$ over $0 \le t \le 1$.
Give *explicit* expressions for $\phi_n(t)$ and λ_n.

(10) b. Write down the optimal detector for deciding H_0 vs. H_1 from $\{R(t), 0 \le t \le 1\}$.

(5) c. Compute d^2 for this detector.

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(10) d. Now we *know* H_1 is true, but A is now an unknown constant. $m = 5.5$ still.
Compute $\hat{A}_{MLE}(\{R(t), 0 \le t \le 1\})$. Simplify as much as possible.

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(10) e. Now we observe $r(t) = \begin{cases} \cos(m\pi t) + n(t) & \text{under } H_1 \\ n(t) & \text{under } H_0 \end{cases}$ over interval $0 \le t \le 1$.

$n(t)$ is from Problem #1. Note $d^2 = 1/2$ for all half-integer m.

An idea strikes us: Let $r'(t) = \int_0^t r(s)ds$ and note $w(t) = \int_0^t n(s)ds$.

Thus we can "integrate" this problem to get the previous problem.

(5) i. Show that by varying m we can make d^2 in the first problem arbitrarily large.

(5) ii. This suggests we can do the same thing in the second problem.
Yet we know $d^2 = 1/2$ for all m! Resolve this apparent contradiction.

WRITE ANSWERS HERE:

(a): $\phi_n(t) =$

$\lambda_n =$

(b):

(c): $d^2 =$

(d): $\hat{A}_{MLE} =$

(e):