PRINT YOUR NAME HERE:

HONOR CODE PLEDGE: "I have neither given nor received aid on this exam, nor have I concealed any violations of the honor code." Open book; SHOW ALL OF YOUR WORK!

SIGN YOUR NAME HERE:

(40) 1. $p_{x|H_0}(X|H_0) = \begin{cases} 2(1-X) & \text{for } 0 \le X \le 1 \\ 0 & \text{otherwise} \end{cases}$ and $p_{x|H_1}(X|H_1) = \begin{cases} 2X & \text{for } 0 \le X \le 1 \\ 0 & \text{otherwise} \end{cases}$.

(5) a. Compute and simplify the likelihood ratio test, using an undetermined threshold.
(5) b. Given a priori probabilities $Pr[H_0] = p$ and $Pr[H_1] = 1 - p$, and MEP criterion, compute the threshold as a function of p for the *simplified* test in (a).
(5) c. Compute closed-form expressions for P_D and P_F in terms of p.
(5) d. Compute a closed-form expression for $P_D(P_F)$. Plot the ROC.
(5) e. Compute the Neyman-Pearson test with level of significance $\alpha = \frac{1}{4}$.
(5) f. Compute the slope of the ROC at this point using (d); show it equals η from (e).
(5) g. Draw a line on your ROC plot specifying the minimax test with MEP criterion.
(5) h. Compute the threshold for the minimax test from (g) (the answer is simple).

WRITE ANSWERS HERE:

(a): $X \gtrless$

(b): $X \gtrless$

(c): $P_D =$

(c): $P_F =$

(d): $P_D =$

(e): $X \gtrless$

(f): slope=

(h): $X \gtrless$

(30) 2. We observe x where $p_{x|a}(X|A) = A^4 X^3 e^{-AX}/6$ for $X \geq 0$; 0 for $X < 0$.

(5) a. Compute $\hat{a}_{MLE}(X)$, the maximum likelihood estimator of A given X.

(5) b. Compute the Cramer-Rao lower bound on any *unbiased* estimator of A.

(5) c. Is $\hat{a}_{MLE}(X)$ an *efficient* estimator? PROVE your answer.

Now we are given that a has *a priori* pdf $p_a(A) = 1/A^2$ for $A \geq 1$; 0 for $A < 1$.

(5) d. Compute $\hat{a}_{MAP}(X)$, the maximum a posteriori probability estimator of a.

(5) e. Compute the Cramer-Rao lower bound on any estimator of random variable a.

(5) f. Is $\hat{a}_{MAP}(X)$ an *efficient* estimator? PROVE your answer.

WRITE ANSWERS HERE:

(a): $\hat{a}_{MLE}(X) =$

(b): $E[(\hat{a} - A)^2] \geq$

(c): YES/NO

(d): $\hat{a}_{MAP}(X) =$

(e): $E[(\hat{a} - a)^2] \geq$

(f): YES/NO

#1:

#2:

#3:

$\sum$:

(30) 3. We have two noisy channels #1 and #2 over which we wish to send YES or NO.
We will send 0.3 on channel #1 and 0.4 on channel #2 to send YES.
We will send 0.4 on channel #1 and 0.3 on channel #2 to send NO.
Channel noises are 0-mean Gaussian with: $E[n_1^2] = \frac{1}{2}$; $E[n_2^2] = \frac{1}{10}$; $E[n_1 n_2] = \frac{1}{5}$.

(10) a. Compute the optimal receiver for deciding whether YES or NO was sent.
Simplify; leave the threshold undetermined for now. Draw a block diagram.

(5) b. Compute the Fisher discriminant d^2. HINT: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} / (ad - bc)$.

(5) c. Compute P_D for the Neyman-Pearson test with level of significance $\alpha = 0.1$.
Do this *graphically* on the ROC curve below; *show your work on it.*

(10) d. Deciding that this P_D is too low, you obtain a third noisy channel,
which has 0-mean Gaussian noise n_3 independent of n_1, n_2 with $E[n_3^2] = 0.12$.
You now send messages $[0.3, 0.4, A]$ for YES and $[0.4, 0.3, -A]$ for NO.
How big must A be to improve P_D at level of significance $\alpha = 0.1$ to $\frac{3}{4}$?

WRITE ANSWERS HERE:

(a):

(b): $d^2 =$

(c): $P_D =$

(d): $A =$