EECS 551 Fall 1997

PROBLEM SET #7

ASSIGNED: October 30, 1997
DUE DATE: November 6, 1997

Read Sections 3.5, 3.6, 4.7, 6.3 (all short) of V&K (mostly 2-D wavelets).
Also skim Chapter 7 of V&K (mostly on image coding, but good background).
This week's theme: Computation of continuous-time wavelet bases and transforms.

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1. *Another* way to derive wavelet scaling functions based on splines:
   Start with the linear spline function $s_1(t) = 1 - |t|, |t| < 1; 0, |t| > 1$.
   a. Compute the sampled autocorrelation $s_3(n)$ of $s_1(t)$ (note $s_3(t) = s_1(t) * s_1(t)$). This consists of exactly three nonzero numbers.
   b. Compute the spectral factorization $S_3(z) = R(z)R(z^{-1})$ of $S_3(z)$. HINT: Use the quadratic formula and write $S_3(z) = c^2(1 - bz^{-1})(1 - bz)$.
   c. Write $\Phi(\omega) = \frac{S_1(e^{j\omega})}{R(e^{-j\omega})}$. NOTE: $\sqrt{3} = 1.732$. Explain why $\phi(t)$ is orthogonal to its translations.
   d. Write $\phi(t) = \sum_{n=0}^{\infty} a_n s_1(t-n)$. Find a_n. What kind of function is $\phi(t)$? HINT: Use $\frac{1}{1-bz^{-1}} = \sum_{n=0}^{\infty} b^n z^{-n}, |b| < 1$ and set $z = e^{j\omega}$.

   NOTE: This is NOT the Battle-Lemarie $\phi(t)$!

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2. Let $x(t) = \begin{cases} e^{-t}, & \text{if } t > 0 \\ 0, & \text{if } t < 0. \end{cases}$
   a. Compute the continuous time Haar wavelet transform of $x(t)$.
   b. Compute the continuous time Haar scaling transform of $x(t)$.
   c. Write out Mallat's fast wavelet algorithm in the general case.
   d. Insert your answers to (a) and (b) in (c) and confirm the algebra works out. NOTE: Make sure you use the correct $h_0(n)$ for Haar (see text p.101). While either $h_0(n)$ or its time reversal can be used in discrete time, consistency with the continuous time Haar requires the right choice.

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3. *Detection of fractal signals in noise using wavelets:*
   Let $x(n)$ be the discrete-time deterministic fractal signal from Problem Set #5.
   Download the MATLAB program from http://www.eecs.umich.edu/aey/eecs551.html.
   Let $y(n) = x(n) +$"50*rand(size(n))" (note this is a LOT of noise!)
   Let $z(n)$ be generated using "-25+100*rand(size(n))" (pure noise signal).
   The numbers are chosen to make $y(n)$ and $z(n)$ have roughly the same size.
   a. PLOT $y(n)$ and $z(n)$ for $1 \leq n \leq 32$. Can you tell which is which?
   b. Run the Haar-filter subband coder on both $y(n)$ and $z(n)$. Look at various outputs and compare them. NOW can you tell which is which? HINT: Compare low-frequency outputs at various scales (rows of y0 and z0) among themselves for $129 \leq n \leq 160$. y0 resemble each other more than z0.

"Mr. Lawyer, what is your fee for answering three questions?" "$1000."
"Isn't that steep?" "Yes. What is your final question?"