1. Wavelets for sparsification of linear systems of equations:
Consider the discrete-time deconvolution problem in which we observe
\( y(n) = h(n) * u(n), 0 \leq n \leq 7 \) and \( h(n) = 0.99^{\lfloor n \rfloor} \) is the impulse response.
We wish to reconstruct \( u(n), 0 \leq n \leq 7 \) from \( y(n), 0 \leq n \leq 7 \) (\( u(n) = 0, n \notin [0,7] \)).
a. Formulate this deconvolution problem as a linear system of equations.
b. Use the Haar basis to get another linear system of equations which is sparse.
A SPARSE linear system matrix has most of its elements very small \( \rightarrow \) negligible.
HINT: \( y = Hx \rightarrow Qy = (QHQ^T)(Qu) \) where \( Q \) is the matrix \( A_0 \) on p.145.
POINT: Sparse systems of equations are easier and faster to solve.

2. Subband Discrete Fourier Transform (DFT):
We want to compute \( 2^N \)-point DFT of \( x(n) \). We know \( X(k) \approx 0 \) for \( k > 2^N-1 \).
We need the higher-order DFT to get good resolution of the frequency components.
However, we can stand to compute only an approximation to \( X(k) \).
b. Preprocess \( x(n) \) by taking its Haar transform: \( H\{x(n)\} = x_1(n) \).
   Show \( X(k) \approx (1 + e^{-j2\pi k/2^n})X_1(k) \) where \( X_1(k) = \text{DFT}[x_1(n)/\sqrt{2}] \).
   Explain why this roughly halves the computation required for \( X(k) \).
c. Now suppose \( X(k) = 0 \) except in a few known frequency bands of interest.
   Show that taking the DFT of a subband decomposition (wavelet transform) of \( x(n) \) saves a substantial amount of computation.

4. Discrete-time fractals:
   A fractal is self-similar:
   \( x(2n) \) and \( x(n) \) "look alike."
The MATLAB program shown at right computes a fractal signal \( x(n) \).
a. PLOT \( x(n), n \in [1,1024] \) and \( [1,128] \).
   It also computes the Haar transform.
as rows of the matrices \( y0 \) and \( y1 \).
b. PLOT the 4th, 5th, 6th rows
   of \( y0 \) on a single plot. Repeat for \( y1 \).
c. Show any wavelet transform of \( x(n) \)
at different scales is almost identical.

Excuse heard in genetic engineering class: "My homework ate the dog."