EECS 551 Fall 1997

PROBLEM SET #3

ASSIGNED: Sept. 18, 1997
DUE DATE: Sept. 25, 1997

Read Section 4.1-4.3 of V&K. Skip Sections 3.4-3.7.
We will do Section 3.6 later during wavelet image processing.
Note similarities between Section 4.1-4.3 and Section 3.1-3.3.
This week's theme: subband coding (don't need Section 4.1-4.3).

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1. The following signal is input into a subband coder:
$u(n) = \{3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9, 3, 2, 3, 8, 4, 6, 2, 6, 4, 3, 3, 8, 3, 2, 7, 9 \ldots\}$
   a. Using $H_0(e^{j\omega}) = (1 + e^{-j\omega})/\sqrt{2}$ in the QMF,
   write out *in the time domain* the signal at each of the first few stages.
   b. Ezplain *in the time domain* why we can downsample without losing anything.
   c. Explain how each stage roughly divides the lowpass output of the previous stage into lowpass and highpass parts, while also achieving time-domain localization.

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2. Using the procedure on p.18 of my notes and p.128-131 of V&K,
"design" the Haar and sinc filters for orthogonal 2-channel filter banks.
For Haar, use $R(z) = 1/2$. For sinc, DON'T perform the spectral factorization.

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3. Some examples of QMFs (orthonormal filters) without multiplication:
   a. Show $|H_0(e^{j\omega})|^2 + |H_0(e^{j(\omega+\pi)})|^2 = 2$ is equivalent to $\sum_i h(i)h(i + 2n) = \delta(n)$.
   HINT: $1 + (-1)^n = 2$ for n even; $= 0$ for n odd.
   b. Show each of the following can be used in a QMF (after normalization):
   $h(n) = \{2, 6, 3, -1\}$; $h(n) = \{4, 16, 16, 0, -4, 1\}$; $h(n) = \{-8, 8, 64, 64, 8, -8, 1, 1\}$

   SIGNIFICANCE: convolution with these can be implemented using bit shifts; no multiplications are required in the subband coder!

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4. The MATLAB program shown at right implements a two-stage subband coder using the Haar filter. It is applied to a simple sinusoidal signal $x(n)$.
   a. Type in and run the program.
   Plot each of the outputs and show on a diagram of the coder where each is.
   b. Interpret your results:
   where is most of the signal?
   c. Show *analytically* why you get what you get in (a).
   d. Now change the sinusoid to $x(n) = \begin{cases} \sin(\frac{\pi}{8}n), & \text{if } n < 0; \\ \sin(\frac{3\pi}{4}n), & \text{if } n > 0 \end{cases}$
   for $-128 < n < 128$. Repeat parts (a),(b).

"A chicken is an egg's way of making another egg"–anonymous