EECS 551 Fall 1997

PROBLEM SET #2

ASSIGNED: Sept. 11, 1997
DUE DATE: Sept. 18, 1997

Read Sections 3.1-3.3 of V&K. You can skip the polyphase parts.
I also won't be using the time-domain matrix presentation in class,
but you may find it useful to see this material from another perspective.

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1. For the signal $x(n)$ with spectrum shown, sketch the spectra at each point:
   i. $x(n) \rightarrow \uparrow 2 \rightarrow \downarrow 3 \rightarrow y(n)$
   ii. $x(n) \rightarrow \downarrow 3 \rightarrow \uparrow 2 \rightarrow y(n)$
   a. Show the end results are identical, but NOT to the original signal.
   b. Show (a) directly using the z-transform, instead of sketching spectra.
   c. Now insert ideal lowpass filters between $\uparrow 2$ and $\downarrow 3$, and between $\downarrow 3$ and $\uparrow 2$. Now decimating and interpolating, not just downsampling and upsampling.
   i. Can we recover $x(n)$?
   ii. Now change from $\uparrow 2, \downarrow 3$ to $\uparrow 3, \downarrow 2$. NOW can we recover $x(n)$?

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2. What if the signal from Problem Set #1 ISN'T bandlimited to 20 kHz?
   Then we need an antialias filter. But that requires capacitors, etc.
   IDEA: Use decimation instead, as follows:
   i. Use a CHEAP antialias filter with transition band 20-140 kHz
   ii. Sample at 160 kHz. Note this will cause aliasing
   iii. Decimate (digital filter then downsample) by 4
   POINT: It is MUCH easier to make the DIGITAL decimation filter sharp
   than to have to make the ANALOG antialias prefilter sharp.

   Sketch a complete system implementing this, and the spectrum at each stage.
   This is actually used in PC sound cards, DAT machines, etc.

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3. Another trick for multirate filtering: do it in stages
   Given a signal bandlimited to 4 kHz, we want only its lowest 75 Hz.
   We want a linear phase FIR filter with the following specs:
   Sampled at F_{sample}=8 kHz. Passband 0-75 Hz. Transition band 75-80 Hz.
   Passband ripple= $\delta_1 = 0.01$. Stopband ripple= $\delta_2 = 0.0001$.
   Kaiser's formula for the filter length is $M = \frac{-10\log_{10}\delta_1\delta_2 - 13}{14.6(F_{stop}-F_{pass})/F_{sample}} + 1$.

a. Compute M for direct implementation (decimate=filter+downsample by 50)?
b. Now decimate in two stages: first by 25, then by 2.
i. For the decimation by 25 the following specs change:
   Transition band now 75 to $\frac{8000}{25} - 80 = 240$ Hz. Sketch spectrum and explain why.
   Passband ripple now $\delta_1 = 0.005$. How long is this FIR filter?
ii. For the decimation by 2 the following specs change:
   Sample rate now $\frac{8000}{25} = 320$ Hz. Passband ripple now $\delta_1 = 0.005$.
   How long is this FIR filter?
c. Show that the total filter length is reduced by a factor of about 13!