SUMMARY OF VARIOUS RELATIONS BETWEEN ANALYSIS AND SYNTHESIS FILTERS IN PERFECT-RECONSTRUCTION FILTER BANKS

Definitions

$H_0(z)$=analysis lowpass filter;$G_0(z)$=synthesis lowpass filter

$H_1(z) = \pm H_0(-z)$=analysis highpass filter

$G_1(z) = \pm G_0(-z)$=synthesis highpass filter

Conditions for Perfect Reconstruction

(my notes p.14; text bottom of p.112)

$G_0(z)H_0(z) + G_1(z)H_1(z) = 2\,(\text{recover}\,x(t))$

$G_0(z)H_0(-z) + G_1(z)H_1(-z) = 0\,(\text{no aliasing})$

BIOrthogonal Filter Banks

(my notes p.15; text bottom of p.121)

$G_0(z) = H_0(z) = H_1(-z); \quad G_1(z) = -H_1(z) = -H_0(-z)$

Watch the signs carefully! Note sign change in HPFs.

These clearly satisfy the "no-aliasing" condition.

The "recover $x(t)$" condition becomes $H_0^2(z) - H_1^2(z) = 2z^{odd}$

Orthogonal Filter Banks

(my notes p.16; text bottom of p.125)

$H_0(z) = G_0(z^{-1}); \quad H_1(z) = G_1(z^{-1})$

Note time reversal between analysis and synthesis.

Link: $G_1(z) = -z^{odd}G_0(-z^{-1})$. Then we have

$H_1(z) = G_1(z^{-1}) = -z^{-odd}G_0(-z) \rightarrow H_1(z) = -z^{-odd}H_0(-z^{-1})$

$\rightarrow H_1(-z) = -(-z)^{-odd}H_0(z^{-1})$

$\rightarrow H_1(-z) = z^{-odd}H_0(z^{-1}) = z^{-odd}G_0(z)$

These clearly satisfy the "no-aliasing" condition:

$G_0(z)H_0(-z) + G_1(z)H_1(-z)$

$= G_0(z)G_0(-z^{-1}) - z^{odd}G_0(-z^{-1})z^{-odd}G_0(z) = 0$

What happened to the sign in the biorthogonal case?
Subtle point: $-(-z)^{-odd} = z^{-odd}$

The "recover $x(t)$" condition becomes either of
$G_0(z)G_0(z^{-1}) + G_0(-z)G_0(-z^{-1}) = 2$
$G_1(z)G_1(z^{-1}) + G_1(-z)G_1(-z^{-1}) = 2$

Defining $P(z) = H_0(z)H_0(z^{-1}) = G_0(z)G_0(z^{-1})$
reduces this to $P(z) + P(-z) = 2$.

Defining $P(z) = (1+z)^K(1+z^{-1})^K R(z)$
and solving linear system of equations (text p.131)
allows us to put K zeros of $G_0(z)$ at the origin.