MULTIRATE FILTERING:  
HOW IT MAKES DSP EASIER

GIVEN: Signal bandlimited to 10kHz, sampled at $f_{Nyquist} = 2(10kHz) = 20kHz$.

WANT: To low-pass-filter (LPF) to 100Hz.

SPECS: $H(f) = \begin{cases} 
1, & \text{if } 0 < |f| < 95Hz; \\
\text{any}, & \text{if } 95Hz < |f| < 100Hz; \\
0, & \text{if } 100Hz < |f| < 10kHz.
\end{cases}$

Note $\frac{\text{transition width}}{\text{maximum freq}} = \frac{5}{10^4}$ → sharp filter.

IDEA:
1. After LPF, can $\downarrow 100$ (since $\frac{10kHz}{100Hz} = 100$).
   Recall LPF followed by $\downarrow 100$ is decimation.
2. If decimate in stages, can use shorter filters → much less computation and storage.
3. For example, decimate by 10 twice:

FIRST STAGE DECIMATION:

$\rightarrow H_1(f) \rightarrow \downarrow 10 \rightarrow$ where $H_1(f) = \begin{cases} 
1, & \text{if } 0 < |f| < 95Hz; \\
\text{any}, & \text{if } 95Hz < |f| < 1905Hz; \\
0, & \text{if } 1905Hz < |f| < 10kHz.
\end{cases}$

1. $\frac{\text{transition width}}{\text{maximum freq}} = \frac{1810}{10^4}$ → dull filter.
2. $1905 = \frac{20kHz}{10} - 95; \quad 1810 = 1905 - 95$
1. Dull filter→short-length FIR filter→ relatively little computation and storage.
2. There is much aliasing, BUT:
3. $0 < |f| < 95\,Hz$ IS NOT ALIASED!
   And $0 < |f| < 95\,Hz$ is all we care about!
   WE DON’T CARE about aliasing the rest!

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**SECOND STAGE DECIMATION:**


to $H_2(f)$ → ↓ 10 → where

$$H_2(f) = \begin{cases} 
1, & \text{if } 0 < |f| < 95\,Hz; \\
n\text{any}, & \text{if } 95\,Hz < |f| < 100\,Hz; \\
0, & \text{if } 100\,Hz < |f| < 1\,kHz.
\end{cases}$$

1. These are same specs as original filter?
   Where are the computational savings?
2. Filter operates on DOWNSAMPLED signal:
   Recall → ↓ 10 → $H_2(z)$ → is equivalent to
   $H_2(z^{10})$ → ↓ 10 (only need every $10^{th}$ output)
3. $\frac{\text{transition width}}{\text{maximum freq}} = \frac{5}{10^3} \rightarrow$ duller than before.
4. Break up into many stages→greater savings
5. Used in digital audio tape, PC sound cards, etc.
   so don’t need sharp analog antialiasing filter.