(40) 1. **Computing continuous-time Haar wavelet transforms:**
   For the function $x(t)$ shown at right:
   
   (10) a. Compute the Haar scaling transform $\bar{x}_j^i$ of $x(t)$.
   (10) b. Compute the Haar wavelet transform $x_j^i$ of $x(t)$.
   Be sure to consider all possible integer values of $i$ and $j \geq 0$.
   (5) c. Write Mallat’s fast wavelet algorithm for the Haar basis.
   (10) d. Insert answers to (a),(b) into (c); confirm algebra works.
   (5) e. For small $i$ $x_j^i$ looks like a scaled Haar basis. Explain this shape.

(40) 2. **Computing 1st-order Battle-Lemarie wavelet transforms:**
Here $n^{th}$-order Battle-Lemarie wavelets are based on $n^{th}$-order spline $s_n(t)$.
Still using the function $x(t)$ from Problem #1:

(5) a. Prove, using subspace ideas ONLY (e.g., $V_i, W_i$), that only scale $i = 0$ is needed to represent $x(t)$ using 1st-order Battle-Lemarie scaling functions.
(10) b. Compute the 1st-order Battle-Lemarie scaling transform of $x(t)$ directly using formula $\bar{x}_j^i = \int x(t)2^{-i/2}\phi(2^{-i}t - j)dt$.
HINT: Write $\phi(t) = \sum c_n s_1(t - n)$; express answer in terms of constants $c_n$.
(10) c. An easier way: Somewhere in the derivation of the 1st-order Battle-Lemarie scaling function there is an equation that gives the answer to (b) directly Compute $DTFT[\bar{x}_j^i]$ as an explicit function of $\omega$.
(10) d. Compute the Fourier transform of the 2nd-order Battle-Lemarie scaling function.
HINTS: $\int s_2(t)dt = \frac{66}{120}$; $\int s_2(t)s_2(t-1)dt = \frac{26}{120}$; $\int s_2(t)s_2(t-2)dt = \frac{1}{120}$.
(5) e. Show that your answer to (d) is piecewise quadratic.

(20) 3. **Computing 2D continuous Haar transforms:**
$z(x, y)$ is defined as $z(x, y) = \begin{cases} 1, x \leq 0, y \geq 0 \\ 3, x \leq 0, y \leq 0 \end{cases}$; $z(x, y) = \begin{cases} 2, x \geq 0, y \geq 0 \\ 4, x \geq 0, y \leq 0 \end{cases}$.

(10) a. Show that the 2D Haar wavelet transform $z_{j,n}^{i,(m)} = 0$ for all $i, j, n$ and $m = 1, 2, 3!$
(5) b. So how does the 2D Haar wavelet transform represent $z(x, y)$?
(5) c. If noise is added to $z(x, y)$, explain how thresholding can remove most of it.

DON’T FORGET TO WRITE OUT AND SIGN THE HONOR PLEDGE:
"I have neither given nor received aid on this exam, nor have I concealed any violation of the Honor Code.”