EECS 551 Fall 1997

EXAM #1

Open book, notes, problem sets. Given under Honor Code. SHOW YOUR WORK! You will be graded on your explanations as well as on your answers.

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(50) 1. Go through all of the wavelet equations for $h_0(n) = \delta(n)$:

(5) a. Show $\{h_0(n-j), j \in \textit{integers}\}$ spans the space of square-summable functions.

(5) b. Write the Smith-Barnwell condition; show this $h_0(n)$ satisfies it.

(5) c. Compute the filters $h_1(n), g_0(n), g_1(n)$.

(5) d. Draw a two-stage-deep subband coder. Specify all filters.

(5) e. Input to (d) is $\{3, 1, 4, 1, 5, 9, 2, 6\}$. Specify signal at each point in the coder.

(5) f. Explain why the time shift in (c) is needed for perfect reconstruction.

(5) g. Starting with $\phi(t) = \begin{cases} 1, & \text{if } 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$

what *exactly* is the k^{th} iteration of the 2-scale equation?

(5) h. Why doesn't the 2-scale equation converge for this $h_0(n)$?

(10) i. The 2-scale equation almost converges to $\phi(t) = \delta(t)$.

Show $\delta(t)$ defines a multiresolution analysis $\cdots V_0 \subset V_{-1} \subset \cdots \neq L^2$

Specify V_0 and give typical nontrivial members of V_0 and V_{-1}.

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(30) 2. Let's design the discrete-time D2 Daubechies basis function *without* performing a spectral factorization (hooray!)

Start by writing $H_0(z) = (1+z)^2(z+b)c$ for some constants b, c.

(5) a. Compute $h_0(n)$ for all n in terms of b, c.

(5) b. Write the Smith-Barnwell condition in the *time domain*.

(5) c. Use (b) to obtain a quadratic equation for b.

(5) d. Solve (c). Explain why you chose the root you chose.

(5) e. Compute $h_0(n)$, leaving c undetermined. Show that it's right!

(5) f. Compute $h_1(n)$ from (e).

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(20) 3. We wish to approximate $\sqrt{|t|}$ over $-1 < t < 1$ using Legendre polynomials $\phi_i(t)$.

Start by writing $\sqrt{|t|} = c_0\phi_0(t) + c_1\phi_1(t) + c_2\phi_2(t)$.

(10) a. Compute c_0, c_1, c_2. NOTE: $\int_{-1}^{1} \sqrt{|t|}t^n \, dt = \begin{cases} \frac{4}{3}, & \text{if } n = 0 \\ 0, & \text{if } n = 1 \\ \frac{4}{7}, & \text{if } n = 2. \end{cases}$

(10) b. Compute a_0, a_1, a_2 minimizing $\int_{-1}^{1} [\sqrt{|t|} - (a_2t^2 + a_1t + a_0)]^2 dt$.

Set $\frac{\partial}{\partial a_i}$[above integral]=0 to get 3 linear equations in 3 unknowns.

Show your answers to (a) and (b) agree. Which way is easier?

DON'T FORGET TO WRITE OUT AND SIGN THE HONOR PLEDGE:

"I have neither given nor received aid on this exam, nor have I concealed any violation of the honor code."