A continuous-time RP \((x(t))\) is separable if the Kolmogorov extension holds for it, i.e., the knowledge of the joint pdf \(f(x_1(t_1), \ldots, x_n(t_n))\) for any times \(t_1, t_2, \ldots, t_n\) and any \(n\) does not completely specify \(x(t)\). Note that all discrete-time RPs are separable.

**Note:** In Exercise 5.1, we may consider separable RPs for an example of a non-separable RP in Exercise 6.

**Exercise:** Increment of a RP over the interval \((t, t + \Delta t)\) is the RV \(X(t + \Delta t) - X(t)\).

**Example:** The increment of a Poisson counting process over \((t, t + \Delta t)\) is the number of arrivals in \([t, t + \Delta t]\).

**Proposition:** If a RP has stationary increments \(f(x(t) - x(t)) = f(x_1(t_1) - x_1(t_1))\) for \((x(t), y(t))\), then the increments are independent over non-overlapping intervals. (Intervals don't overlap if their intersection is \(\emptyset\).)

**Exercise:** (i) Poisson counting process (ii) Wiener process (iii) Many types of "quantum" processes

**Theorem:** For any non-decreasing sequence \(\{\tau_n\}\) of stopping times, \(\left(\tau_n\right)\) is a separable RP if and only if \(\tau_n\) is bounded away from \(\infty\) and \(\lim_{n \to \infty} \tau_n = \infty\).

**Proof:**
1. **Lemma 1:** If \(X(t)\) is a separable RP, then \(X(t)\) is separable for any \(t > 0\), and \(X(\cdot)\) is separable if \(X(t)\) is separable for any \(t > 0\).
2. **Lemma 2:** If \(X(t)\) is a separable RP, then the increments \(X(t + \Delta t) - X(t)\) are independent of \(X(t)\) for any \(t > 0\).
3. **Lemma 3:** If \(X(t)\) is a separable RP, then the increments \(X(t + \Delta t) - X(t)\) are independent of \(X(t)\) for any \(t > 0\).

**Theorem:** Assume that \(X(t)\) is a separable RP, then the increments \(X(t + \Delta t) - X(t)\) are independent of \(X(t)\) for any \(t > 0\).

**Proof:**
1. Assume \(X(t)\) is a separable RP, then the increments \(X(t + \Delta t) - X(t)\) are independent of \(X(t)\) for any \(t > 0\).
2. Assume \(X(t)\) is a separable RP, then the increments \(X(t + \Delta t) - X(t)\) are independent of \(X(t)\) for any \(t > 0\).
3. Assume \(X(t)\) is a separable RP, then the increments \(X(t + \Delta t) - X(t)\) are independent of \(X(t)\) for any \(t > 0\).

**Corollary:** Using the above, we can show directly that if \(X(t)\) is an RP, then \(X(t)\) is separable for any \(t > 0\).

**Proof:** Recall that \(X(t)\) is an RP, then \(X(t)\) is separable for any \(t > 0\).