**DEF:** A *continuous-time random process* $x(t)$ is a mapping $x:\Omega\to\mathcal{R}^{\mathcal{R}}$, or: $x(t,\omega):(\mathcal{R}\times\Omega)\to\mathcal{R}$ where $\Omega$=sample space and $\mathcal{R}=\{reals\}$.

1. Fix $t_o\in\mathcal{R}\to x(t_o,\omega)$=random variable indexed by time index $t_o$.
2. Fix $\omega_o\in\Omega\to x(t,\omega_o)$=sample function=realization (not continuous).
3. Kolmogorov Extension Thm *no longer holds:* $\exists$*unmeasurable* rps.

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**DEF:** $x(t)$ is $N^{th}$*-order stationary* if joint pdfs of order $N$ have: $f_{x(t_1)\dots x(t_N)}(X_1\dots X_N)=f_{x(t_1+\tau)\dots x(t_N+\tau)}(X_1\dots X_N)$ for any $\tau$.

**DEF:** $x(t)$ SSS *strict sense stationary* $\Leftrightarrow N^{th}$-order stationary for all $N$.

**Note:** iid$\to$SSS$\to N^{th}$-order$\to 2^{nd}$-order$\to$WSS$\to 1^{st}$-order$\leftrightarrow$id.

**DEF:** $x(t)$ *Gaussian* $\leftrightarrow\{x(t_1),x(t_2)\dots x(t_N)\}$ JGRV for all $t_1\dots t_N$.

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**DEF:** $x(t)$ WSS *wide sense stationary* $\Leftrightarrow \mu(t)=\mu$ and $K_x(t,s)=K_x(t-s)$.

**where:** *(Auto)covariance* $K_x(t,s)=E[x(t)x(s)]-E[x(t)]E[x(s)]=\lambda_{x(t),x(s)}$.

**Props:** (1) $K_x(0)=\sigma^2_{x(t)}\geq 0$; (2) $K_x(\tau)=K_x(-\tau)$; (3) $|K_x(\tau)|\leq K_x(0)$.

**DEF:** $x(t)$ *white* $\Leftrightarrow x(t)$ WSS and $K_x(t,s)=\sigma^2\delta(t-s)\Leftrightarrow S_x(\omega)=\sigma^2$.

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$$E[y(t)]=\int_{-\infty}^{\infty}h(t-s)E[x(s)]ds=\int_{-\infty}^{\infty}h(s)E[x(t-s)]ds \quad (h(t)\text{=impulse response}).$$

$$K_{xy}(t,s)=\int_{-\infty}^{\infty}h(u)K_x(t,s-u)du=\int_{-\infty}^{\infty}h(s-u)K_x(t,u)du=K_{yx}(s,t).$$

$$K_y(t,s)=\int\int h(t-u)h(s-v)K_x(u,v)du\,dv=\int\int h(u)h(v)K_x(t-u,s-v)du\,dv.$$

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**WSS:** $E[y(t)]=\mu\int_{-\infty}^{\infty}h(s)ds=H(0)\mu$=constant.

**WSS:** $K_y(t,s)=\int\int h(u)h(v)K_x((t-u)-(s-v))du\,dv$
$=\int\int h(u)h(v)K_x((t-s)-u+v)du\,dv=K_y(t-s)\to y(t)$ also WSS.

**WSS:** $K_{xy}(t,s)=\int h(u)K_x(t-s+u)du=K_{xy}(t-s)\to x,y$ jointly WSS.

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**Transfer function:** $H(\omega)=\int_{-\infty}^{\infty}h(t)e^{-j\omega t}dt\Leftrightarrow h(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}H(\omega)e^{j\omega t}d\omega$.

**PSD:** $S_x(\omega)=\int_{-\infty}^{\infty}R_x(\tau)e^{-j\omega\tau}d\tau=2\int_0^{\infty}R_x(\tau)\cos(\omega\tau)d\tau$ ($R_x(\tau)$ even).

**Then:** $S_y(\omega)=H(\omega)H(-j\omega)S_x(\omega)=|H(\omega)|^2S_x(\omega)$. Much more later!

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**EX1:** *Wiener* process=0-mean,II,Gaussian process$\Leftrightarrow$cont-time random walk.

**EX2:** *Poisson* counting process=II; Poisson increments$\Leftrightarrow$#arrivals in $[0,t]$.

**EX3:** *WGN* (White Gassian Noise)=white Gaussian rp (need not be noise).

**EX4:** WGN$\to$ $\left|\frac{dy}{dt}+ay=bu\right|$ $\to$Gauss-Markov rp ($1^{st}$-order system only).