1a. $E[k]=\frac{n}{2}$. $\sigma_k^2=n\frac{1}{2}(1-\frac{1}{2})=\frac{n}{4}$. $\sigma_k=\sqrt{n}/2$. **(b):** $1600(\frac{1}{2})=800$.

1b. $Pr[790\leq k\leq 810]=\Phi[\frac{810-800}{\sqrt{1600}/2}]-\Phi[\frac{790-800}{\sqrt{1600}/2}]=2\Phi[0.5]-1=0.383$.

1c. $0.9=Pr[k\geq b]=1-\Phi[\frac{b-800}{20}]\rightarrow\frac{800-b}{20}=\Phi^{-1}[0.9]=1.28\rightarrow b=774$.

1d. $0.95=Pr[k\geq 1000]=1-\Phi[\frac{1000-\frac{n}{2}}{\sqrt{n}/2}]$ $\qquad b=800-20(1.28)=774$.

$\rightarrow\frac{\frac{n}{2}-1000}{\sqrt{n}/2}=\Phi^{-1}[0.95]=1.645\rightarrow n-1.645\sqrt{n}-2000=0$.

1e. Solving $(\sqrt{n})^2-1.645\sqrt{n}-2000=0\rightarrow n=2075$ is the positive root.

1f. $e=\frac{k}{n}-\frac{1}{2}\rightarrow\sigma_e^2=\frac{1}{n^2}\sigma_k^2=\frac{1}{4n}$. $\underset{n\rightarrow\infty}{\text{LIM}}\frac{1}{4n}=0$. Note $\frac{1}{2}$ has no effect.

2a. $[1,1,1]'$ associated with 0 eigenvalue$\rightarrow x_1+x_2+x_3=0\rightarrow x_3=-5$.

2b. $f_{x_1,x_3}(X_1,X_3)\sim\mathcal{N}\left(\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}2&-1\\-1&2\end{bmatrix}\right)$.

2c. $\sigma_y^2=[1,2,3]K[1,2,3]'=[1,2,3][-3,0,3]'=6$.

2d. $\lambda_{x_1x_2}^2\leq\sigma_{x_1}^2\sigma_{x_2}^2\rightarrow 1=(-1)^2\leq(2)(2)=4$ checks.

2e. $z=e_2\cdot x$ and $w=e_3\cdot x\rightarrow z,w$ independent$\rightarrow z\sim\mathcal{N}(0,6)$,

where $\sigma_z^2=\sigma_{x_1}^2+\sigma_{x_2}^2-2\lambda_{x_1,x_2}=2+2-2(-1)=6$ (or do as in (c)).

2f. $(\hat{x_1})_{LSE}(X_3)=\frac{\lambda_{x_1x_3}}{\sigma_{x_3}^2}X_3=-\frac{1}{2}X_3$ since all 0-mean.

3a. $p_{r|a}(R|A)=\binom{N}{R}A^R(1-A)^{N-R}\rightarrow\hat{a}_{MLE}(R)=\frac{R}{N}$ from lecture.

3b. $0=\frac{\partial}{\partial A}[\log p_{r|a}(R|A)+\log f_a(A)]=\frac{R}{A}-\frac{N-R}{1-A}-10$

$\rightarrow 10A^2-(10+N)A+R=0$. Solve for A.

3c. Solving $10A^2-(92+10)A+20=0\rightarrow A=10,\frac{1}{5}\rightarrow\hat{a}_{MAP}(20)=1/5$.

3d. $\hat{a}_{LSE}(R)=\frac{\int Af_{r|a}(R|A)f_a(A)dA}{\int f_{r|a}(R|A)f_a(A)dA}=\frac{\frac{1}{4}(1/4)^R(3/4)^{N-R}\frac{1}{3}+\frac{1}{2}(1/2)^R(1/2)^{N-R}\frac{2}{3}}{(1/4)^R(3/4)^{N-R}\frac{1}{3}+(1/2)^R(1/2)^{N-R}\frac{2}{3}}$

3e. $\frac{\frac{1}{4}(1/4)^2(3/4)^1\frac{1}{3}+\frac{1}{2}(1/2)^3\frac{2}{3}}{(1/4)^2(3/4)^1\frac{1}{3}+(1/2)^3\frac{2}{3}}=\frac{3+32}{12+64}=35/76$.

3f. $E[a]=\frac{1}{3}(\frac{1}{4})+\frac{2}{3}(\frac{1}{2})=\frac{5}{12}$. $E[r]=E_a[r|a]=\frac{1}{3}(144)\frac{1}{4}+\frac{2}{3}(144)\frac{1}{2}=60$.

$E[ar]=E_aE[ar|a]=\frac{1}{3}(144)(\frac{1}{4})^2+\frac{2}{3}(144)(\frac{1}{2})^2=27$.

$\lambda_{ar}=E[ar]-E[a]E[r]=27-(\frac{5}{12})(60)=2$. **Not required:** $\sigma_r^2=$

$E[\sigma_{r|a}^2]+Var[E[r|a]]=\frac{1}{3}(144)\frac{1}{4}(1-\frac{1}{4})+\frac{2}{3}(144)\frac{1}{2}(1-\frac{1}{2})+144^2\sigma_a^2=321$.

3g. $\hat{a}_{LLSE}(R)=\frac{5}{12}+\frac{2}{321}(R-60)=0.0428+0.00623R$.